1 INTRODUCTION

REWRITINGS are archetypal form of all COMPUTATIONS

For examples, all structures such that

- Finite Automata
- Turing Machines
- Grammars
- Petri Nets ...

are, in fact — in a more or less hidden mode — varied forms of REWRITING. see [Bu 02]

This examples are only
low dimension Machines.

We can imagine highest
dimension similar Machines
like as Automata and Grammars for
n-dimensional objects
(for n = 2, images with pixels)

Here We limit ourselve to the
two low dimension cases:

- Rewriting of WORDS (n=1)
- Rewriting of TERMS (n=2)

[plus a generalization of this
last for n = 3]

Here, we will suppose that we know notions as

- categories
- functors
- natural transformations
- left adjoints

and familiar categories as

- $\underline{Ens}$ = category of sets
- $\underline{Graph}$ = " " graphs
- $\underline{Cat}$ = " " categories

and, also, underlying canonical functors, as for example

$$U : \underline{Cat} \longrightarrow \underline{Grph}$$

The mathematical structure

wich is at the FOUNDATION

of all this rewriting forms

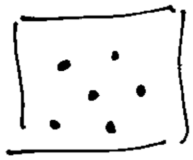
are the POLYGRAPHS

(also called : computads)

or n-polygraphs $(n \in \mathbb{N})$.

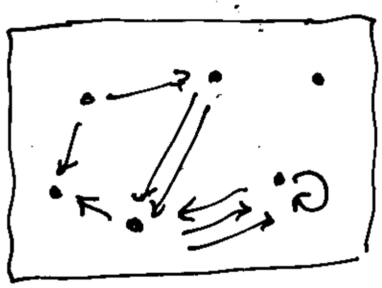
For $n = 0$,

0-polygraphs = sets.



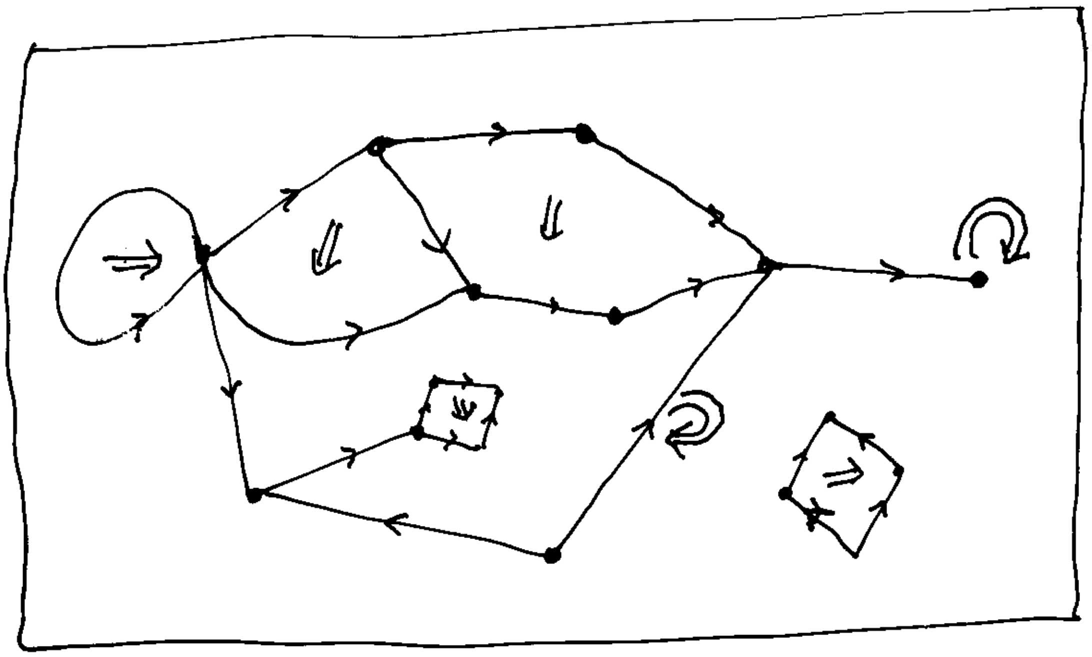
For $n = 1$,

1-polygraphs = graphs.



For $n = 2$,

2-polygraphs are graphs plus 2-cells between paths of same source and target



The *n-graphs* (also called "globular sets") are diagramms G of the form

$$G_0 \underset{t_0}{\overset{s_0}{\leftleftarrows}} G_1 \underset{t_1}{\overset{s_1}{\leftleftarrows}} G_2 \quad \cdots G_{n-1} \underset{t_{n-1}}{\overset{s_{n-1}}{\leftleftarrows}} G_n$$

With
$$s_{i-1} s_i = s_{i-1} t_i \qquad (1 \le i < n-1)$$
$$t_{i-1} s_i = t_{i-1} t_i$$

They form a category

$\boxed{\underline{\mathrm{Grph}}_n}$ = category of n-graphs

(Homomorphism: $f : G \to G'$

$$\begin{array}{ccccccccc}
G_0 & \leftleftarrows & G_1 & \leftleftarrows & G_2 & \cdots & G_{n-1} & \leftleftarrows & G_n \\
{\scriptstyle f_0}\downarrow & & {\scriptstyle f_1}\downarrow & & {\scriptstyle f_2}\downarrow & & {\scriptstyle f_{n-1}}\downarrow & & {\scriptstyle f_n}\downarrow \\
G'_0 & \leftleftarrows & G'_1 & \leftleftarrows & & & G'_{n-1} & \leftleftarrows & G'_n
\end{array}$$
)

The n-categories (strict) with n-functors form a category

$$\boxed{\underline{Cat}_n} = \text{category of } n\text{-categories}$$

We know

Compositions, identities.

$$\cdot \xleftarrow{v} \cdot \xleftarrow{u} \cdot \qquad x \cdot \circlearrowleft \mathrm{id}^1(x)$$

$$= \cdot \xleftarrow{v *_0 u} \cdot$$

$$\begin{tikzcd} \cdot & \cdot \arrow[l, bend right] \arrow[l, bend left] \arrow[l, phantom, "\Downarrow y"] & \cdot \arrow[l, bend right] \arrow[l, bend left] \arrow[l, phantom, "\Downarrow x"] \end{tikzcd} \qquad \begin{tikzcd} \cdot & \cdot \arrow[l, "u"'] \end{tikzcd} \text{ with } \mathrm{id}^2(u)$$

$$= \begin{tikzcd} \cdot & \cdot \arrow[l, bend right] \arrow[l, bend left] \arrow[l, phantom, "\Downarrow y *_0 x"] \end{tikzcd}$$

$$\vdots$$

$$\begin{tikzcd} \cdot & \cdot \arrow[l, bend right=60] \arrow[l] \arrow[l, bend left=60] \arrow[l, phantom, shift right=2, "\Downarrow x"] \arrow[l, phantom, shift left=2, "\Downarrow x'"] \end{tikzcd}$$

$$= \begin{tikzcd} \cdot & \cdot \arrow[l, bend right] \arrow[l, bend left] \arrow[l, phantom, "x' *_1 x"] \end{tikzcd}$$

$\cdots$ etc...

We have <u>underlying functors</u>

$$\underline{Cat}_n \xrightarrow{U_n} \underline{Gph}_n$$

which have <u>left adjoints</u>

$$\underline{Cat}_n \xleftarrow{L_n} \underline{Gph}_n$$

But, in general, we will note for every n-graph G

$$G^* = L_n(G)$$

n-graph are special cases of n-polygraphs that we will define now.

Description of n-polygraphs

$$
\begin{tikzcd}
P_0 \arrow[d, equal] & P_1 \arrow[dl, shift right, "s_0"'] \arrow[dl, shift left, "t_0"] \arrow[d, "i_1"] & P_2 \arrow[dl, shift right, "s_1"'] \arrow[dl, shift left, "t_1"] \arrow[d, "i_2"] & P_3 \arrow[dl, shift right, "s_2"'] \arrow[dl, shift left, "t_2"] \arrow[d, "i_3"] & \cdots \\
P_0^* & P_1^* \arrow[l, shift right, "\bar{s}_0"'] \arrow[l, shift left, "\bar{t}_0"] & P_2^* \arrow[l, shift right, "\bar{s}_1"'] \arrow[l, shift left, "\bar{t}_1"] & P_3^* \arrow[l, shift right, "\bar{s}_2"'] \arrow[l, shift left, "\bar{t}_2"] & \cdots
\end{tikzcd}
$$

(See more explanations in [Bu 93])

But we can define more abstractly and globaly the category $\underline{\mathrm{Pol}}_n$ of n-polygraphs.

This definition is inductive with $n \in \mathbb{N}$.

Before, we need to define another category

$$\boxed{\underline{\mathrm{Cat}}_n^+}$$

which is the category of $(n+1)$-**marked categories**.

A $(n+1)$-marked categories is a n-category ~~with~~ which has in addition $(n+1)$-cells, but without identities or compositions for such $(n+1)$-cell.

This category is defined by a pullback of underlying functors

$$\begin{tikzcd} \underline{Cat}_n \arrow[d] & \underline{Cat}_n^+ \arrow[l] \arrow[d] \arrow[dl, phantom, "\lrcorner"] \\ \underline{Grph}_n & \underline{Grph}_{n+1} \arrow[l] \end{tikzcd}$$

But the comparison of this square with the commutative square

$$\begin{tikzcd} \underline{Cat}_n \arrow[r] \arrow[d] & \underline{Cat}_{n+1} \arrow[d] \\ \underline{Grph}_n & \underline{Grph}_{n+1} \arrow[l] \end{tikzcd}$$

gives a functor evident

$$W_{n+1} : \underline{Cat}_{n+1} \longrightarrow \underline{Cat}_n^+ .$$

This functor forgets the compositions of $(n+1)$-cells (and identities data).

This last functor admits a left adjoint

$$K_{n+1} : \underline{Cat}_n^+ \longrightarrow \underline{Cat}_{n+1}$$

which gives formal $(n+1)$-identities and $(n+1)$-compositions.

For example, for $n = 0$

$$\underline{Cat}_0^+ = \underline{Grph}$$

and the functor

$$K_1 : \underline{Cat}_0^+ \longrightarrow \underline{Cat}$$

is the functor "free category generated by a graph".

Now, $\boxed{\underline{\mathrm{Pol}}_n}$, the category of n-polygraphs is defined recursively and simultaneously with a functor

$$\overline{L}_n : \underline{\mathrm{Pol}}_n \longrightarrow \underline{\mathrm{Cat}}_n$$

$P^* = \overline{L}_n(P)$ will be the "free category" generated by the n-polygraph P)

To begin

for $n = 0$ $\quad \underline{\mathrm{Pol}}_0 = \underline{\mathrm{Ens}}$.

and
$$\overline{L}_0 : \underline{\mathrm{Pol}}_0 \longrightarrow \underline{\mathrm{Cat}}_0 = \underline{\mathrm{Ens}}$$
is the identity functor ($X^* = X$).

Next we define $\overline{L}_{n+1}$ from $\overline{L}_n$ by the pullback

$$\begin{tikzcd} \underline{Pol}_n \arrow[d] & \underline{Pol}_{n+1} \arrow[l] \arrow[d] \arrow[dl, phantom, "\lrcorner"] \\ \underline{Grph}_n & \underline{Grph}_{n+1} \arrow[l] \end{tikzcd}$$

(which defines also $\underline{Pol}_{n+1}$ ...)

where $\underline{Pol}_n \longrightarrow \underline{Grph}_n$ is factorized like this

$$\begin{tikzcd} \underline{Pol}_n \arrow[rr] \arrow[dr, "\overline{L}_n"'] & = & \underline{Grph}_n \\ & \underline{Cat}_n \arrow[ur, "\text{(underlying functor)}"'] & \end{tikzcd}$$

And a second factorization gives $\overline{L}_{n+1}$

$$\begin{tikzcd} \underline{Pol}_{n+1} \arrow[rr, "\overline{L}_{n+1}"] \arrow[dr] & = & \underline{Cat}_{n+1} \\ & \underline{Cat}_n^+ \arrow[ur, "K_{n+1}"'] & \end{tikzcd}$$

2 LAWVERE THEORY as 2-MONOIDS

In an equationnal system, such that Lawvere theory presented by a signature and equationnal axioms, the rewriting of terms

is not,

a priori, a rewriting in a polygraph, because:

When we construct terms, we use cartesian products and their universal property.

Recall the definition

A <u>cartesian category</u> is a category which admits binary cartesian products and final objet, that is to say, which satisfy this axiom:

$$\boxed{\begin{array}{l}
\forall (X \quad Y) \quad \exists (X \leftarrow P \rightarrow Y) \\[2ex]
\forall (X \leftarrow P' \rightarrow Y) \quad \exists \left(\begin{tikzcd} & P' \arrow[dl] \arrow[dr] \arrow[d, "m"] & \\ X \arrow[r, phantom, "="] & P \arrow[l] \arrow[r] \arrow[r, phantom, "="] & Y \end{tikzcd} \right) \\[2ex]
\forall \left(\begin{tikzcd} & P' \arrow[dl] \arrow[dr] \arrow[d, "m'"] & \\ X \arrow[r, phantom, "="] & P \arrow[l] \arrow[r] \arrow[r, phantom, "="] & Y \end{tikzcd} \right) \quad (m = m')
\end{array}}$$

and an analogous for final objet.

This is far of equationnal axiom like

$$\forall \ldots \quad (m = m')$$

This definition of cartesian category is not suitable here for two reasons:

1) This definition is given by <u>property</u> and not by <u>data</u>

2) The axiom is far to be <u>equationnal</u> ($\forall\exists\forall\exists\forall \ldots$).

We have showed [Burroni 93] that it is possible to eliminate universal property for the benefit of an equationnal system in all cartesian category.

Then, we reformulate :

A <u>cartesian category</u> is a category C with following <u>data</u> :

(i) A monoidal structure on C :

$$C = (C, 1, \times)$$

(for simplicity, we will suppose that the monoidal structures are always <u>stricts</u>)

(ii) We have two natural transformations ε, δ.
Precisely, this gives, for all objets X, Y of C, two morphisms

$$1 \xleftarrow{\varepsilon X} X \xrightarrow{\delta X} X^2 = X \times X$$

(and their are natural in X).

and

(iii) we must satisfy two equations

(*) $$\boxed{\varepsilon 1 = id(1)}$$

(* *) for all X, Y in C

$$\boxed{(X \times \varepsilon Y) \times (\varepsilon X \times Y) \circ \delta(X \times Y) = id(X \times Y)}$$

($X \times \varepsilon Y : X \times Y \to X$, $\varepsilon X \times Y : X \times Y \to Y$ are the canonical projection of the cartesian product $X \times Y$)

$$\begin{tikzcd}
& (X \times Y)^2 \arrow[d, bend right=60, "(X\times\varepsilon Y)\times(\varepsilon X\times Y)"'] & \\
& X \times Y \arrow[u, "\delta(X \times Y)"'] \arrow[dl, "X \times \varepsilon Y"'] \arrow[dr, "\varepsilon X \times Y"] & \\
X & & Y
\end{tikzcd}$$

We deduce of this, for all X in $\mathcal{C}$:

$$\boxed{\begin{aligned} (X \times \varepsilon X) \circ \delta X &= id(X) \\ (\varepsilon X \times X) \circ \delta X &= id(X) \end{aligned}}$$

[It is a remark of Y. Lafont that this formulas, primary considered as axioms, can be deduced of (**) for $Y = 1$ or $X = 1$]

In diagrammatic form

$$\begin{tikzcd}
 & X \times Y \arrow[dr, "(X\times\varepsilon Y)\times(\varepsilon X\times Y)"] \arrow[d, phantom, "="] & \\
X \times Y \arrow[ur, "\delta(X\times Y)"] \arrow[rr, "id(X\times Y)"'] & \, & X \times Y
\end{tikzcd}$$

$$\begin{tikzcd}
 & X^2 \arrow[dr, "X\times\varepsilon X"] \arrow[d, phantom, "="] & \\
X \arrow[ur, "\delta(X)"] \arrow[rr, "id(X)"] \arrow[dr, "\delta(X)"'] & \, \arrow[d, phantom, "="] & X \\
 & X^2 \arrow[ur, "\varepsilon X\times X"'] &
\end{tikzcd}$$

Now, a Lawvere theory

is a cartesian category T

in which there is a generating

object $\underline{S}$ of T

That is to say, every object

of T is of the form

$$\underline{S}^n = \underline{S} \times \underline{S} \times \cdots \times \underline{S} \qquad (n \in \mathbb{N})$$

(in particular $\underline{S}^0 = 1$, $\underline{S}^1 = \underline{S}$)

A T-algebra is a

functor

$$\theta : T \longrightarrow \underline{\mathrm{Ens}}$$

which preserves products;

$A = \theta(\underline{S})$ is the underlying set

and we note $\theta = (A, \theta)$.

But, with our new point of view on cartesianity, we can consider Lawvere theory as a

strict monoidal category
with a cartesian structure

$$T = (T, 1, \times, \varepsilon, \delta).$$

The same is true for the category of sets

$$\underline{\mathrm{Ens}}(T, 1, \times, \varepsilon, \delta),$$

if we consider — as usually do all algebraists (classical algebraists) — the monoidal law $\times$ as like if there strict.

A T-algebra is reduced to as a homomorphism of <u>2-monoid</u>

$$\theta : (T, 1, \times) \to (\underline{\mathrm{Ens}}, 1, \times)$$

<u>(2-monoid = strict monoidal categorie)</u>

We can now state [Burroni 93]:

A Lawvere theory T <u>finitely presented</u> by an equationnal System (Ω, E) can be <u>equivalently</u> presented by a <u>finite 3-polygraph</u>

$$P = P(\Omega, E)$$

Ω = (family of operations of the form $\alpha : \underline{S}^n \to \underline{S}$)

E = (family of equations of the form $\lambda = \lambda'$ where $\lambda, \lambda' : \underline{S}^m \rightrightarrows \underline{S}$ are derived Terms of Ω)

Before explaining general cases, I want to illustrate with the simplest case:

T_{set} = Lawvere <u>theory of sets</u>

that is to say:

- <u>no</u> operation $\Omega = \phi$
- <u>no</u> equation $E = \phi$.

This is the <u>initial</u> Lawvere theory.

But the associated 3-polygraph

$$P = P(\phi, \phi)$$

is not empty.

<u>générator</u> of T_{set}:

$P_0 = \{ * \}$
$P_1 = \{ \underline{s} \}$
$P_2 = \{ \varepsilon, \delta, \sigma \}$ $\quad 1 \xleftarrow{\varepsilon} \underline{s} \xrightarrow{\delta} \underline{s}^2 \circlearrowleft \sigma$

And $P_3 = \{10 \text{ equations}\}$

that I will give further.

In fact we have an isorphism

$$\boxed{T_{set} \simeq \mathcal{F}^{op}}$$

where $\mathcal{F}$ is the category of finite set or, rather, category of integer where, for all $n \in \mathbb{N}$ we set

$$n = \{0, 1, \dots, n-1\}$$

$\mathcal{F}$ with coproduct is a <u>strict monoidal category</u>:

$$\mathcal{F} = (\mathcal{F}, 0, +)$$

where 0 is <u>initial object</u>.

<u>Lemma</u> $\mathcal{F}$, as 2-monoid (= 2-category with one 0-cell : $*$), is finitely generated by three 2-cells:

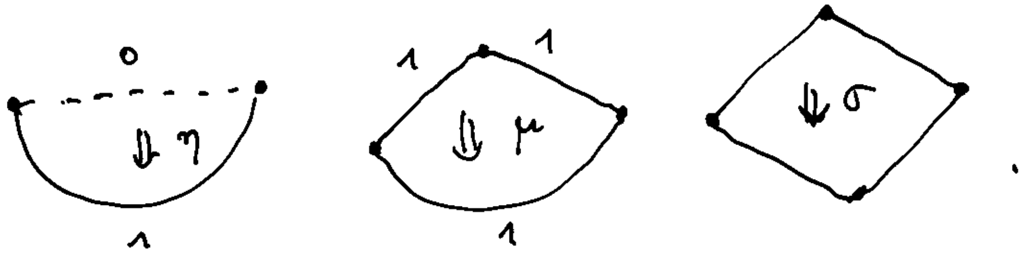
$$\eta : 0 \to 1 \ , \quad \mu : 2 \to 1 \ , \quad \sigma : 2 \to 2$$

$$\mu(0) = \mu(1) = 0 \qquad \begin{matrix} \sigma(0) = 1 \\ \sigma(1) = 0 \end{matrix}$$

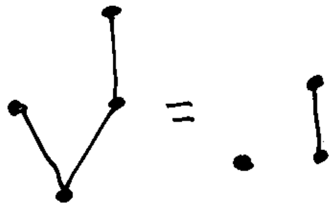
Symbolically

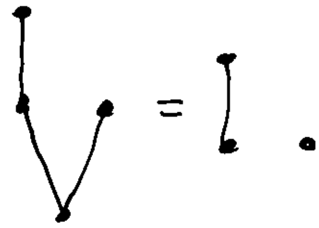
$$\eta = (\,\cdot\,) \ , \quad \mu = (\bigvee) \ , \quad \tau = (\times)$$

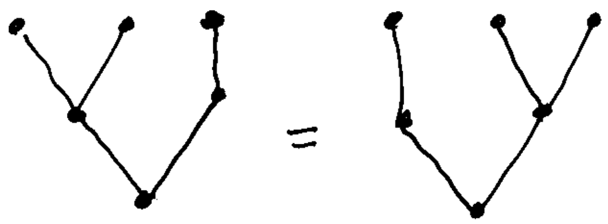
or, in polygraph style

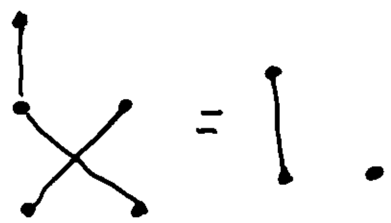


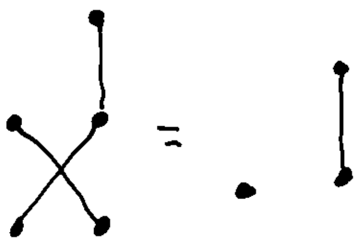
Now, Here is the promise 10 equations
(But Yves Lafont has given system more compacted)

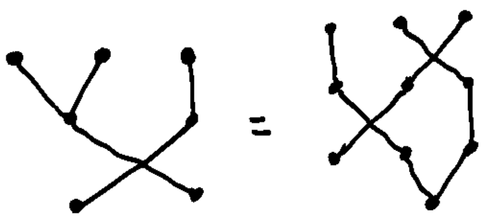
(1) 

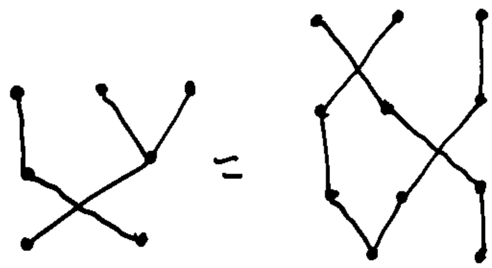
(2) 

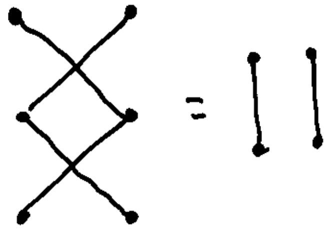
(3) 

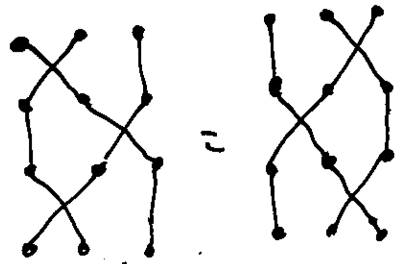
(4) 

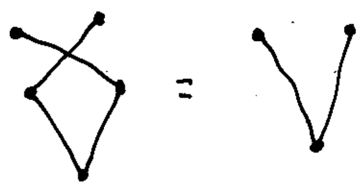
(5) 

(6) 

(7) 

(8) 

(9) 

(10) 

When we change

$$\mathcal{F} \text{ into } \mathcal{F}^{op} = T_{set}$$

we change vocabulary

$$0 \rightsquigarrow 1$$

$$1 \rightsquigarrow \underline{S}$$

$$2 \rightsquigarrow \underline{S}^2$$

$$\vdots$$

$$+ \rightsquigarrow \times$$

$$\eta \rightsquigarrow \varepsilon$$

$$\mu \rightsquigarrow \delta$$

$$\tau \rightsquigarrow \sigma$$

and the 1-composition (i.e. composition with contact of 1-dimension) is multiplicatively noted

$$p \xrightarrow{u} q \xrightarrow{v} r = p \xrightarrow{vu} r$$

<u>Lemma</u> *

$$T_{set} = (\mathcal{F}^{op}, 1, \times)$$

as 2-monoid is finitely presented

by the 3-polygraph $P(\phi, \phi)$:

$$\begin{tikzcd}
\{*\} \arrow[d, equal] & \{\underline{s}\} \arrow[dl, shift left] \arrow[dl, shift right] \arrow[d] & \{\varepsilon, \delta, \sigma\} \arrow[dl, shift right, "s_2"'] \arrow[dl, shift left, "t_2"] \arrow[d] & \{\overset{10}{\text{equations}}\} \arrow[dl, shift right, "s_3"'] \arrow[dl, shift left, "t_3"] \\
\{*\} & \{\underline{s}^n \mid n \in \mathbb{N}\} \arrow[l, shift left] \arrow[l, shift right] & \{\varepsilon, \delta, \sigma\}^* &
\end{tikzcd}$$

The 4-dimensional term is intended for quotients.

<u>Remark</u>. By convention

$$s_3(\lambda = \lambda') = \lambda \qquad t_3(\lambda = \lambda') = \lambda$$

for every equation $\lambda = \lambda'$.

Now, we consider a general equational system (finite) (Ω, E). The 4-polygraph has the form

$$\begin{tikzcd}
\{*\} \arrow[d, equal] & \{\underline{s}\} \arrow[dl, shift left] \arrow[dl, shift right] \arrow[d] & \bar{\Omega} \arrow[dl, shift left, "s_2"'] \arrow[dl, shift right, "t_2"] \arrow[d] & \bar{E} \arrow[dl, shift left, "s_3"'] \arrow[dl, shift right, "t_3"] \\
\{*\} & \{\underline{s}^n \mid n \in \mathbb{N}\} & \bar{\Omega}^* &
\end{tikzcd}$$

with

- for all $\alpha : \underline{s}^n \to \underline{s}$ in Ω, we have $\alpha \in \bar{\Omega}$ with $s_2(\alpha) = \underline{s}^n$, $t_2(\alpha) = \underline{s}$.

- $\varepsilon, \delta,$ $\in \bar{\Omega}$ with $s_2(\varepsilon) = \underline{s}$, $t_2(\varepsilon) = 1$,
  $s_2(\delta) = \underline{s}$, $t_2(\delta) = \underline{s}^2$,
  $s_2(\sigma) = \underline{s}^2 = t_2(\sigma)$.

- for all $\lambda = \lambda' : \underline{s}^n \Rightarrow \underline{s}$ in E we have $(\lambda = \lambda') \in \bar{\Omega}$ with
  $s_2(\lambda = \lambda') = \lambda$
  $t_2(\lambda = \lambda') = \lambda'$

- The "10 equations", or rather their duals, are in $\bar{E}$ with evident source and target.

- And $3k$ equations (where $k = \#(\Omega)$) to ensure that the 3 2-cells

$$\varepsilon,\ \delta,\ \sigma$$

are extended in natural transformations.

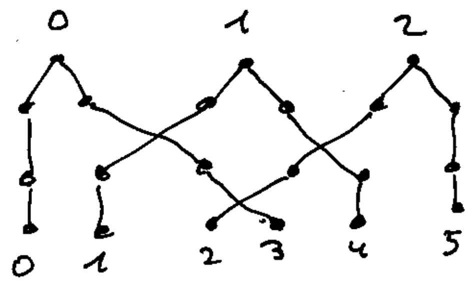
<u>Example</u>: for an operations $\alpha : \underline{S}^3 \to \underline{S}$,

the square

$$\begin{tikzcd}
\underline{S}^6 \arrow[r, "\alpha\times\alpha"] \arrow[d, "\delta_3"'] \arrow[dr, phantom, "="] & \underline{S}^2 \arrow[d, "\delta = \delta_1"] \\
\underline{S}^3 \arrow[r, "\alpha"'] & \underline{S}
\end{tikzcd}$$

is commutative, where

$$\delta_3 = (\underline{S}^2 \times \sigma \times \underline{S}^2)(\underline{S} \times \sigma^2 \times \underline{S})\,\delta^3$$

$=$ 

(see [Bu 93])

(3) 2-Lawvere theories as 3-monoids

We can go further in the description of equational structures of higher dimensions presented as n-monoids.

Before, let us return to a classical example of Lawvere theory which will enlighten our generalisation.

let

$$T_{mon} = \underline{\text{Theory of monoid}}$$

be the simplest example.

We have two operations

$$\begin{tikzcd} 1 \arrow[r, "e"] & \underline{S} & \underline{S}^2 \arrow[l, "m"'] \end{tikzcd}$$

and three equations, for example the associativity

$$\begin{tikzcd} \underline{S} & \underline{S}^2 \arrow[l, "m"'] & \underline{S}^3 \arrow[l, shift right, "m \times \underline{S}"'] \arrow[l, shift left, "\underline{S} \times m"] \arrow[ll, bend left=40, "m(m \times S)"] \arrow[ll, bend left=70, "m(S \times m)"'] \end{tikzcd}$$

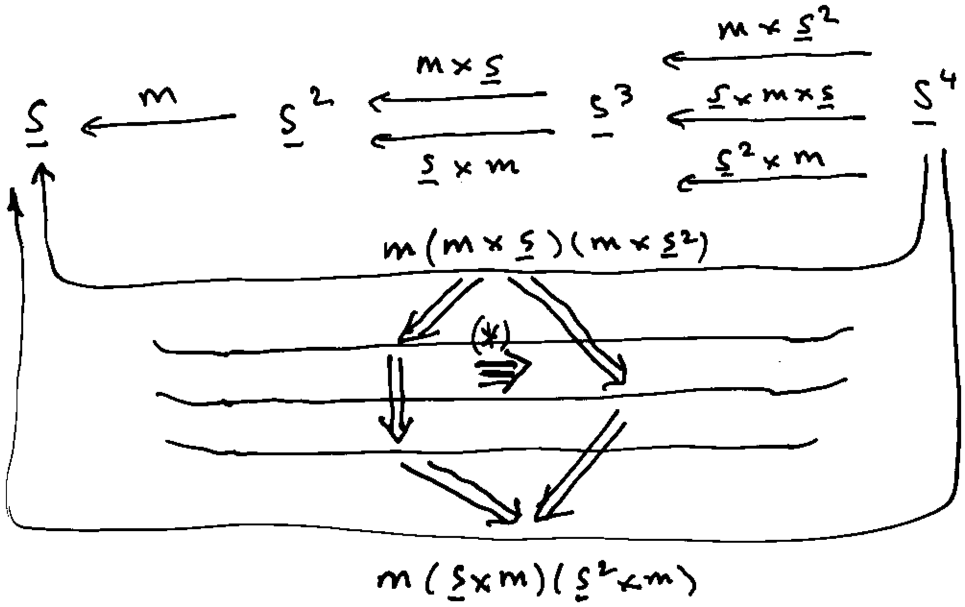
$$=$$

If we replace this axiom by a 3-cell

$$\begin{tikzcd} \underline{S} & \underline{S}^2 \arrow[l, "m"'] & \underline{S}^3 \arrow[l, shift right, "m \times \underline{S}"'] \arrow[l, shift left, "\underline{S} \times m"] \arrow[ll, bend left=40, "m(m \times \underline{S})"] \arrow[ll, bend left=70, "m(\underline{S} \times m)"'] \end{tikzcd}$$

$$\Downarrow a$$

which not collapses, but is submits to a coherence axiom expressed by a 4-cell ("<u>Mac Lane pentagon</u>"):



In detail

(*)

$$
\begin{tikzcd}[column sep=small]
 & m(m\times \underline{s})(m\times \underline{s}^2) \arrow[dl, Rightarrow] \arrow[dr, Rightarrow] & \\
m(m\times \underline{s})(\underline{s}\times m\times \underline{s}) \arrow[d, Rightarrow] & \text{(Mac Lane Pentagon)} \quad \Rrightarrow & m(m\times m) \arrow[ddl, Rightarrow] \\
m(\underline{s}\times m)(\underline{s}\times m\times \underline{s}) \arrow[dr, Rightarrow] & & \\
 & m(\underline{s}\times m)(\underline{s}^2\times m) &
\end{tikzcd}
$$

Remark. The i-cells in this diagramme are, in fact, (i+1)-cells in the 3-monoid.

With this new data, T_{mon} gives a 3-monoid.

$$T_{lax\text{-}mon}$$

which is the <u>2-Lawvere theory</u> of <u>lax-monoidal category</u>.

But, to make sense this must be realized not in $\underline{Ens} = (\underline{Ens}, 1, \times)$ but in the <u>cartesian 2-category</u>

$$\underset{\sim}{Cat} = (\underline{Cat}, 1, \times).$$

There is many other examples of 2-Lawvere theory

- monoidal symmetric categories
- " braided "
- cartesian categories

⋮

But, this is nothing less than a begining.

REFERENCES

[Bu 93] Higher dimensional word problem with application to equational logic.

Theor. Comp. Sci. 115, 43-62, Elsevier 1993

[Bu 02] Automata and Grammars of higher dimensions

(Manuscript) Ecole thematique : Marseille, Luminy. Fev. 2002

http://www.math.jussieu.fr/~burroni/