The Blind Spot

Lectures on Logic, Rome, Autumn 2004

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This series of lectures on proof-theory is *a priori* dedicated to mathematicians and computer-scientists, physicists, philosophers and linguists ; and, since we are no longer in the XVIþ— not to speak of the XVIIIþ — century, it is doomed to failure. In contrast to a course focusing on subdomains which work well (model-theory, set-theory), not that well (temporal or modal logics), or not at all (quantum or epistemic logics), and which would therefore be grounded a certain technical excellency, or, more prosaically, on a well-understood circle of scientific welfare. This being said, plain success is not the only possible goal ; mine might simply be the exposition of a disorder in this apparently well-organised universe, in which logic eventually took its place between two beer mugs and the *Reader's Digest*, and does not disturb, no longer disturbs — a sort of fat cat purring on the carpet.

At the eve of last century, the cat was rather a wolf-dog, the strongly barking kind ; the XXþcentury has been the century of totalitarisms of every possible kind, in particular the linguistic totalitarism (rebaptised « turn »). This extreme form of *scientism* which consisted in the reduction of any mathematical question (and, therefore, everything being supposedly mathematisable, any question) to a question of formal, linguistic, bureaucratic, protocols : Kafka was waiting behind the door. Far from literature, the same scientism was involved, as early as 1904, in the improvement of the human species in Namibia, imagine all these gallows in a country without trees ! Modern logic remains basically impregnated with the « 1900 spirit », this sort of pretension to simplify everything, since one can solve everything. When, after 1930, incompleteness will shake this haughtiness, one will observe hardly more than a complexification of the discourse : instead of explaining from the simpler, one will explain from the « meta » : begins the time of counterfeit coinage. This is also since that time that logic, unable to operate its own counter-reformation, will sever the links with mathematics, physics, etc.

A typical sophism : what is the point of seeking beautiful mathematical structures for logic ? Such a thing cannot exist, since, as mathematics, good or bad, can be translated in logic, the logical structure must reflect the worse, i.e., not to exist, or, at least, remain very bleak. For instance, when looking for a topological, continuous, interpretation of logic, one will head for the worse (e.g., Scott domains), and one will even be proud of this ! Among the revealing details, this insistence of logicians to choose counter-intuitive symbols, in order to make sure that one does not suggest that certain properties — say distributivity — might be more important than others[1] : « More important, really ? How do you define importance ? ». This reminds me of my daughter Isabelle — then very young — « Why not calling the door "spoon" and the

[1]Witness for instance, at the eve of linear logic, the point of honour taken by those who insisted to note « par » + and « with » ×, while « par » distributes over « times ».

spoon "door" ? », to which I answered « When one says "Make for the door", this should not be taken as invitation to supper ». Among the magisterial mistakes of logic, one will first mention quantum logic, whose ridiculousness can only be ascribed to a feeling of superiority of the language — and ideas, even bad, as soon as they take a written form — over the physical world. Quantum logic is indeed a sort of punishment inflicted to nature, guilty of not obeying to the prejudices of logicians... this reminds one of Xerxes having the sea — who had destroyed a boat bridge — whipped.

One century ago, very scarce were those daring to oppose scientist certainties. After one century of slaughters, this is much easier : even if the same baloney sempiternally comes back, like the intelligent robot, fantasy of artificial intelligence and unlikely prosthesis for those who badly need it, we have won the right to make fun of scientific Jivaros. For instance of this H. Simon, the guy who had his computer rediscover Kepler's third law (squares and cubes), forgetting that it is not the law linking the period and the semimajor axis which is hard to find, it is the very idea of such a law, especially for an... astrologer like Kepler.

It would be fair to observe that, in spite of its heavy scientist liabilities, the balance of logic, although limited, is not empty. Model-theory et set-theory are doing rather well ; even proof-theory has a non-negligible balance, et, by the way, what would I otherwise start from, since my topic will essentially be *proof-theory* ?

In the beginning of last century, Einstein's relativity, and, in a more radical way, quantum physics, called our « fundamental intuitions » in question again. Logic, because of its excesses, decided to run into emptiness ; the non-structure, the non-significant « Everything can be coded in everything[2], 1885), and also of the idea of translating images into sound, or rather gurglings ! ». Still, in the « linguistic turn », the idea of pregnancy of the language was deeply inspired, and didn't deserve to become this « machine à décerveler[3] » that we just mentioned. With a closer look, the pregnancy of language contains the germ of another form of « relativisation », in fact of *derealisation* of nature. This is the viewpoint I will try to develop.

A last word : one has the right to find this project crazy, and prefer people starting with something like « *A language is a finite alphabet with whom one constructs terms, formulas, proofs — syntax —; the language is in turn interpreted in a model — semantics —; eventually, this is formalised in a metasystem.* ». But then one does not do logic, at least no foundations, why not sealing in the Behring strait ? The domain, as it ossified during the XXþcen-

[2]And conversely, I suppose ! The idea of mutual codings is ancient and universal : think of des Esseintes and his *orgue à liqueurs* (Huysmans, *À rebours*

[3]To remove the brain.

tury is everything but crazy : a cemetery of ideas. In other terms, the only excuse in the XXIþcentury for indulging in « foundations », is the « grain de folie », i.e., the slight madness.

About the title : it is while revising the text (Summer 2005) that I noticed the recurrence of the expression « blind spot ». The blind spot, this is what one does not (is not) see(n), and one does not even know that one does not see it[4]. The most trivial blind spot, its is when a bad modal logic is justified by a bad Kripke semantics, and *vice versa* ; but one find similar blindings in the most elaborated interpretations. The good news of these lectures, is that the *procedural* standpoint seems to be capable of dislodging the unsaid, the unseen. Simply, while the absence of *Hauptsatz* is enough to show that logic **S5** is wrong, one has to work much more to imagine what could be false in the principles justifying — say — the function 2^n.

Bibliography : [49].

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Three months of lecturing together with the simultaneous composition of lectures notes of 500-odd pages, this is very heavy, it takes six days a week. I would never have succeeded without the support of the audience, not very numerous, but fervent. Nor without the constant attention of Louise.

Two additional subsections, 15.C and 15.D have been written for the essential by Olivier Laurent. The definite version takes into account corrections suggested by Thomas Streicher (chapter 2), and of the long list of the typos found by Akim Demaille.

[4]Kreisel in 1984, speaking of certain Americans : « They have no soul and they don't know that they have no soul ».