Concurrent Nets
A Study of Prefixing in Process Calculi

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Concurrency and Mobility ™

Concurrency and Mobility ™

concurrency CCS

Concurrency and Mobility ™

$$
\begin{array}{ccccc}
\text{concurrency} & & \text{CCS} & & \\
\downarrow & & \downarrow & & \\
\text{mobility} & & \pi\text{-calculus} & \longleftarrow & \lambda\text{-calculus}
\end{array}
$$

Concurrency and Mobility ™

concurrency	CCS	
↓	↓	
mobility	π-calculus	⟵ λ-calculus
	↓	
geometric communication	fusion calculus	

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geometric prefixing	concurrent nets		

Concurrency and Mobility ™

concurrency	CCS		
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geometric communication	fusion calculus	⟹	solos calculus no prefixing
	↓		
geometric prefixing	concurrent nets		

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Syntax

We assume a numerable set of names:

channels $u, v, x, y \in \mathcal{C}$

Agents are defined as:

terms	$P := \mathbf{0} \mid (P \mid P) \mid \nu x.P \mid \quad \alpha$
actions	$\alpha := \bar{u}(\tilde{x}) \mid u(\tilde{x})$

This is the calculus of solos.

Syntax

We assume two disjoint sets of names:

channels	$u, v, x, y \in \mathcal{C}$
locations	$\ell, m \in \mathcal{L}$

Agents are defined as:

terms	$P := \mathbf{0} \mid (P \mid P) \mid \nu x.P \mid \ell : \alpha \mid \langle \pi \rangle\, P$
actions	$\alpha := \bar{u}(\tilde{x}) \mid u(\tilde{x})$
prefixes (guards)	$\pi := 0 \mid 1 \mid \ell \mid \pi + \pi \mid \pi\pi$

This is the calculus of concurrent nets.

Structural rules

- standard rules for scoping and α-conversion
- $(\Pi, 0, +)$ and $(\Pi, 1, .)$ are commutative monoids, mutually distributive, with

$$1 + x = 1 \qquad\qquad 0\,x = 0$$

- **distribution** of prefixes:

$$\langle \pi \rangle\,(P \mid Q) \equiv \langle \pi \rangle\,P \mid \langle \pi \rangle\,Q$$
$$\langle \pi \rangle\,\nu x.P \equiv \nu x.\,\langle \pi \rangle\,P$$

- **composition**:

$$\langle 0 \rangle\,P \equiv \mathbf{0} \qquad \text{nullity}$$
$$\langle 1 \rangle\,P \equiv P \qquad \text{neutrality}$$
$$\langle \pi_1 \rangle\,\langle \pi_2 \rangle\,P \equiv \langle \pi_1 \pi_2 \rangle\,P \qquad \text{composition}$$

Reduction (simplified)

The reduction is that of fusion/solos:

$$u(\tilde{x}) \mid \bar{u}(\tilde{y}) \mid P \longrightarrow P[\tilde{x} = \tilde{y}]$$

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Except that using $\ell : \alpha$ sets the semaphore ℓ to 1, which may free the guards of other actions.

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A small example:

$$\ell : a(x) \mid m : b(y) \mid \langle \ell m + n \rangle \, \bar{c}(xy) \mid \bar{a}(u) \mid \bar{b}(v) \mid n : d(z)$$

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A small example:

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A small example:

$$\bar{c}(uv) \mid n : d(z)$$

The LTS — communication rules

Two kinds of labelled transitions:

$\nu\tilde{x}.\alpha, L$	emit α, open the scopes of $\tilde{x}$, free the guards in L
φ, L	apply the fusion φ, free the guards in L

Actions are emitted as

$$\frac{}{\ell : \alpha \xrightarrow{\alpha,\{\ell\}} \mathbf{0}}$$

and synchronised as

$$\frac{P_1 \xrightarrow{\nu\tilde{x}_1.\bar{a}(\tilde{y}_1),L_1} P_1' \qquad P_2 \xrightarrow{\nu\tilde{x}_2.a(\tilde{y}_2),L_2} P_2' \qquad |\tilde{y}_1| = |\tilde{y}_2|}{P_1 \mid P_2 \xrightarrow{\{\tilde{y}_1=\tilde{y}_2\}\setminus\tilde{x}_1\tilde{x}_2,L_1\cup L_2} \nu\tilde{x}_1\tilde{x}_2.(P_1' \mid P_2')\sigma[1/L_1, L_2]}$$

where σ implements $\{\tilde{y}_1 = \tilde{y}_2\}$ and $z \notin \tilde{x}_1\tilde{x}_2 \Rightarrow \sigma(z) \notin \tilde{x}_1\tilde{x}_2$

The LTS — contextual rules

actions | *fusions*

Scoping:

$$\frac{P \xrightarrow{\nu\tilde{x}.a^{\varepsilon}(\tilde{y}),L} P' \qquad z \notin a\tilde{y}}{\nu z.P \xrightarrow{\nu\tilde{x}.a^{\varepsilon}(\tilde{y}),L} \nu z.P'} \qquad \frac{P \xrightarrow{\varphi,L} P' \qquad z \notin \varphi}{\nu z.P \xrightarrow{\varphi,L} \nu z.P'}$$

$$\frac{P \xrightarrow{\nu\tilde{x}.a^{\varepsilon}(\tilde{y}),L} P' \qquad z \in \tilde{y},\ z \neq a}{\nu z.P \xrightarrow{\nu z\tilde{x}.a^{\varepsilon}(\tilde{y}),L} P'} \qquad \frac{P \xrightarrow{\varphi,L} P' \qquad z\ \varphi\ y,\ z \neq y}{\nu z.P \xrightarrow{\varphi\setminus\{z\},L} P'[y/z]}$$

Composition:

$$\frac{P \xrightarrow{\nu\tilde{x}.\alpha,L} P'}{P \mid Q \xrightarrow{\nu\tilde{x}.\alpha,L} P' \mid Q[1/L]} \qquad \frac{P \xrightarrow{\varphi,L} P'}{P \mid Q \xrightarrow{\varphi,L} P' \mid Q[1/L]}$$

Algebraic formulation

Processes can be written in normal form:

$$P \equiv \nu\tilde{y}.\prod_{i=1}^{n} \langle \pi_i \rangle\, \ell_i : u_i^{\varepsilon}(\tilde{x}_i)$$

with prefixes normalised as

$$\pi_i \equiv \sum_{j=1}^{p_i} \ell_{i,j,1} \cdots \ell_{i,j,q_{i,j}}$$

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channels: $\mathcal{C}$ = free names of P
interface: $\mathcal{I} = \mathcal{C} \setminus \tilde{y}$
locations: $\mathcal{L} = \{\ell_i \mid 1 \leqslant i \leqslant n\}$
actions: $\{+,-\} \times \mathcal{C} \times \mathcal{C}^*$

with prefixes normalised as

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for each ℓ_i, p_i sets of locations

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$$\pi_i \equiv \sum_{j=1}^{p_i} \ell_{i,j,1} \cdots \ell_{i,j,q_{i,j}}$$

enabling relation: $\vdash \subset \mathcal{P}(\mathcal{L}) \times \mathcal{L}$

Graphical syntax

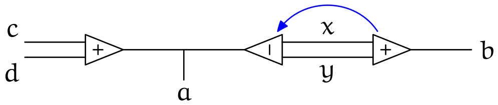
Consider the process

$$\bar{a}(cd) \mid \nu xy.a(xy).\bar{b}(xy)$$

this is written

$$\bar{a}(cd) \mid \nu xy.\big(\ell : a(xy) \mid \langle \ell \rangle\ \bar{b}(xy)\big)$$

and represented as



Conjunction and disjunction

The process

$$\nu xy.\big(\ell : a(x) \mid m : b(y) \mid \langle \ell m \rangle\, \bar{c}(xy)\big)$$

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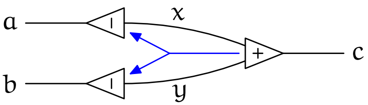

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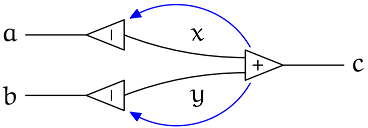
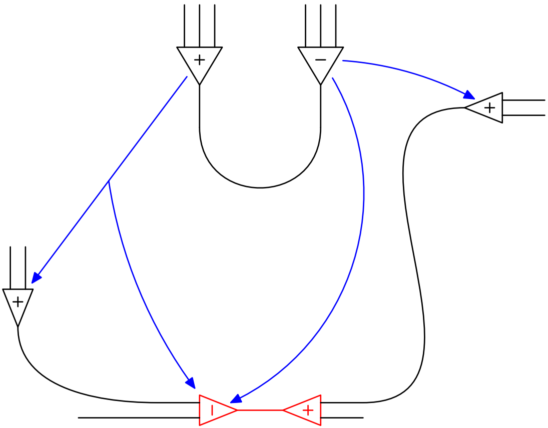


Reduction II



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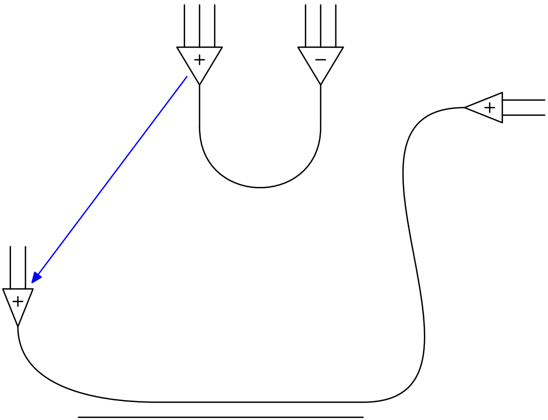

Solos

The agents of the calculus of solos are defined as

$$P := \mathbf{0} \mid (P \mid P) \mid \nu x.P \mid \bar{u}(\tilde{x}) \mid u(\tilde{x})$$

This is the fragment without guards.

The fusion calculus

The agents of the fusion calculus are defined as:

$$\begin{aligned} P, Q := \ & \mathbf{0} \\ \mid \ & P \mid Q \\ \mid \ & \nu x.P \\ \mid \ & \bar{u}(\tilde{x}).P \\ \mid \ & u(\tilde{x}).P \end{aligned}$$

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\end{aligned}
$$

P is translated as $[\![P]\!]_1$ where $[\![P]\!]_\ell$ is defined as:

$$
\begin{aligned}
& \mathbf{0} \\
& [\![P]\!]_\ell \mid [\![Q]\!]_\ell \\
& \nu x.[\![P]\!]_\ell \\
& \langle \ell \rangle\, m : \bar{u}(\tilde{x}) \mid [\![P]\!]_m \\
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\end{array}
$$

This is the fragment with *simple and acyclic prefixing*:

- all prefixes have the form $\langle \ell \rangle\, P$ or $\langle 1 \rangle\, P$
- the relation $\vdash$ is acyclic

The π-calculus

The agents of the fusion calculus are defined as:

$$
\begin{aligned}
P, Q := \;& \mathbf{0} \\
\mid\;& P \mid Q \\
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\mid\;& \bar{u}\langle\tilde{x}\rangle.P \\
\mid\;& u(\tilde{x}).P
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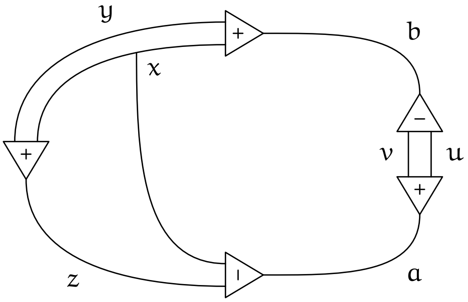
& \nu x.[\![P]\!]_\ell \\
& \langle\ell\rangle\, m : \bar{u}(\tilde{x}) \mid [\![P]\!]_m \\
& \nu\tilde{x}.(\langle\ell\rangle\, m : u(\tilde{x}) \mid [\![P]\!]_m)
\end{aligned}
$$

- it is a sub-calculus of fusion
- receptions are binders and prefixes

Causality: an example

$$P := a(yx) \mid \bar{z}(xy) \mid \bar{b}(yx) \mid b(uv) \mid \bar{a}(uv)$$



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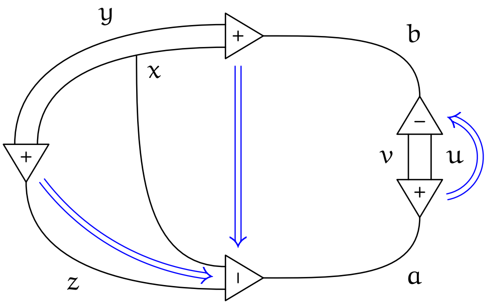

Causality: an example

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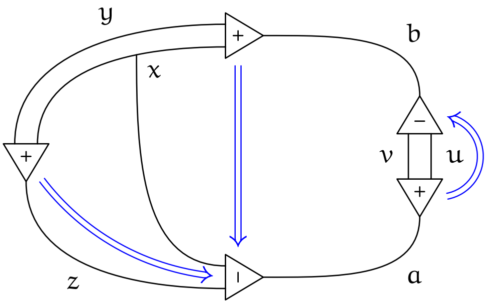

$$P \simeq \left[\!\left[a(yx).\left(\bar{z}\langle xy\rangle \mid \bar{b}\langle yx\rangle\right) \mid b(uv).\bar{a}\langle uv\rangle \right]\!\right]_\pi$$

Causality: another example

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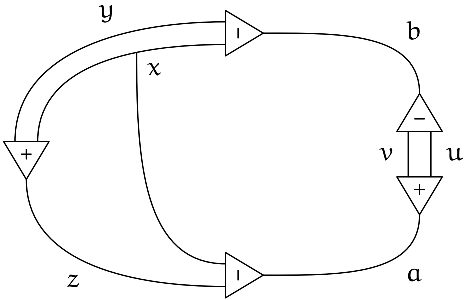

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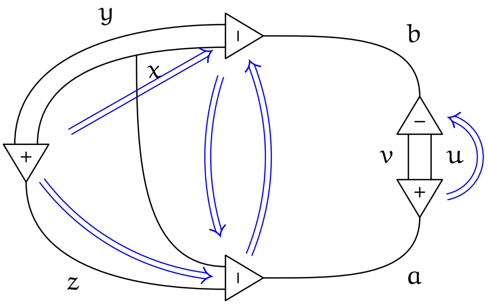

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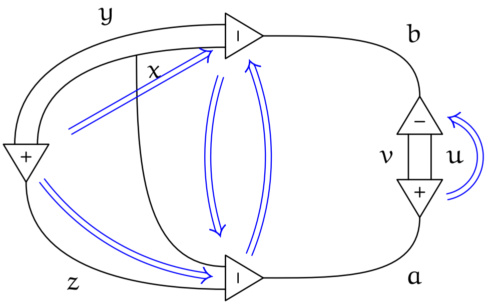


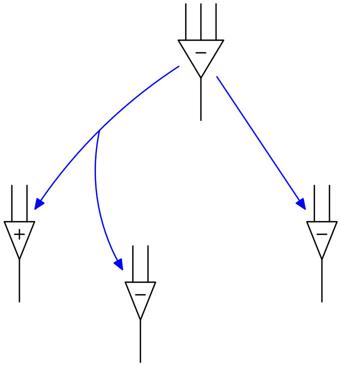
P cannot be a term of π-calculus

Prefixing vs Causality

- Guards are explicit sequentiality constraints, communication implies implicit ones.
- Laneve & Victor: encoding of fusion-calculus in the fragment without prefixes.
- What we do next: encodings of guards.

monotonic guards $\longleftrightarrow$ monotonicity of fusion

Expansive translation



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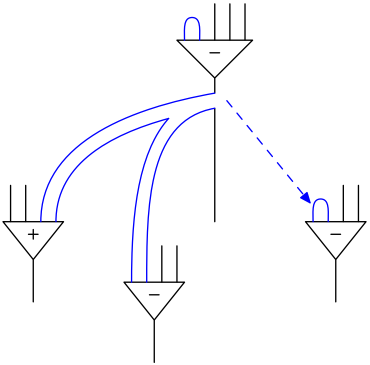

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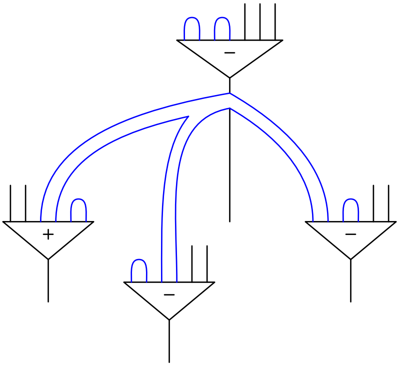
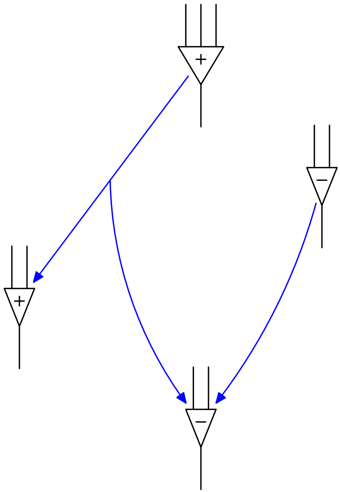


Duo translation



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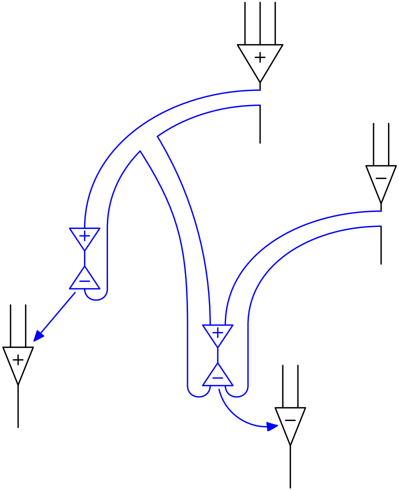

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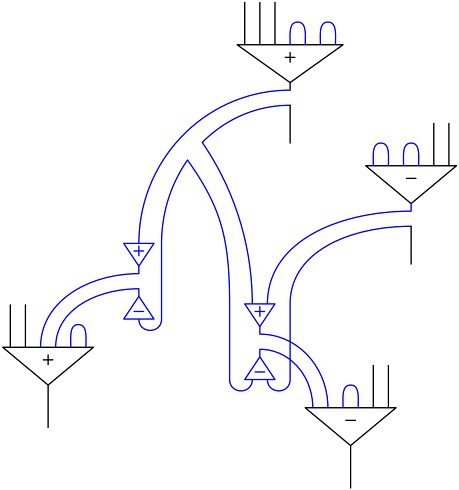


Conclusion

What we have:

- Geometric flavour of prefixing.
- Communication by unification is expressive enough to encode arbitrary monotonic guards.
- All of this can be extended with replication.

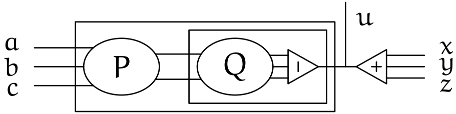
What next?

- Non-monotonic guards are strictly more expressive (external sums).
- What part of communication is *not* a form of prefixing?

Thanks for coming.

The Hidden Slide

Replication can be implemented using boxes:



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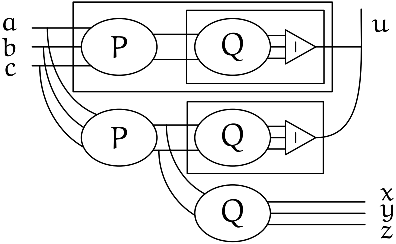

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