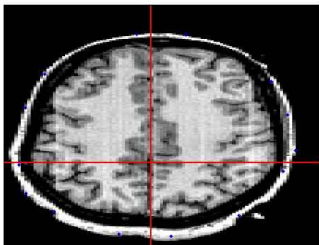
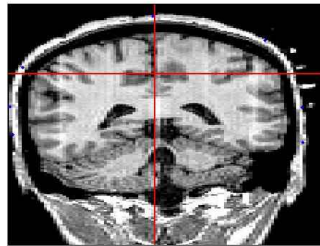
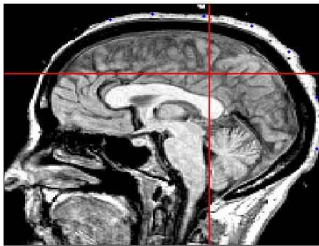
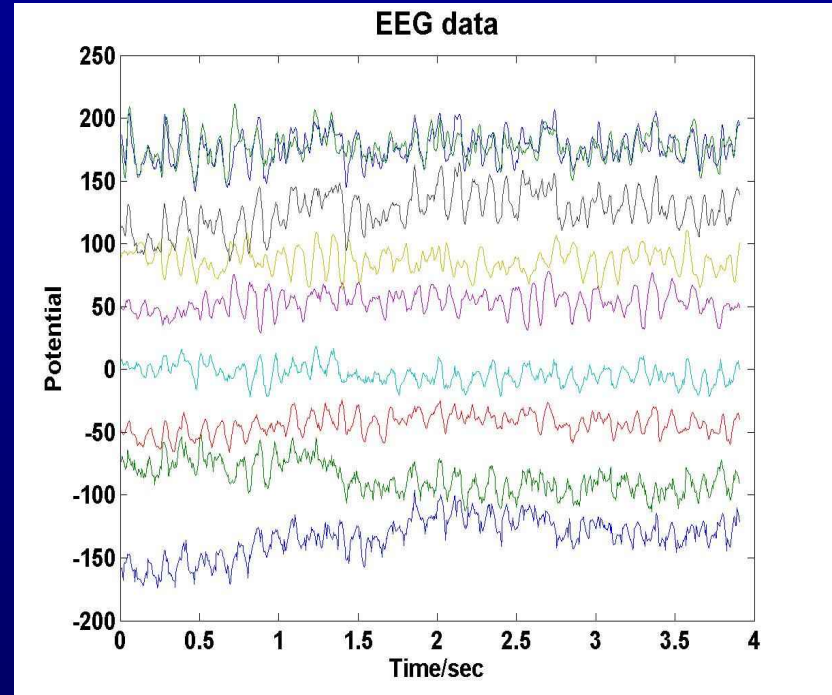
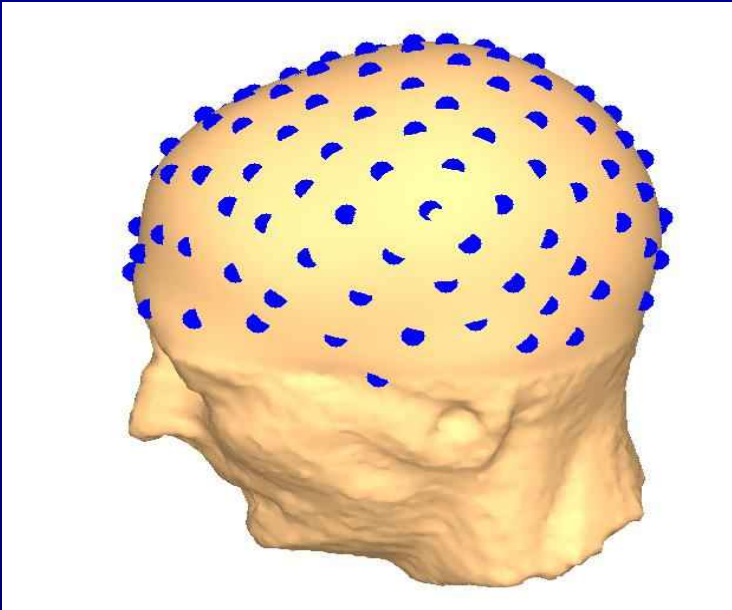


***Estimation of causal direction from
time series in the presence of mixed
and colored noise***

G. Nolte

Fraunhofer FIRST, Berlin

EEG



Typical Properties of Data

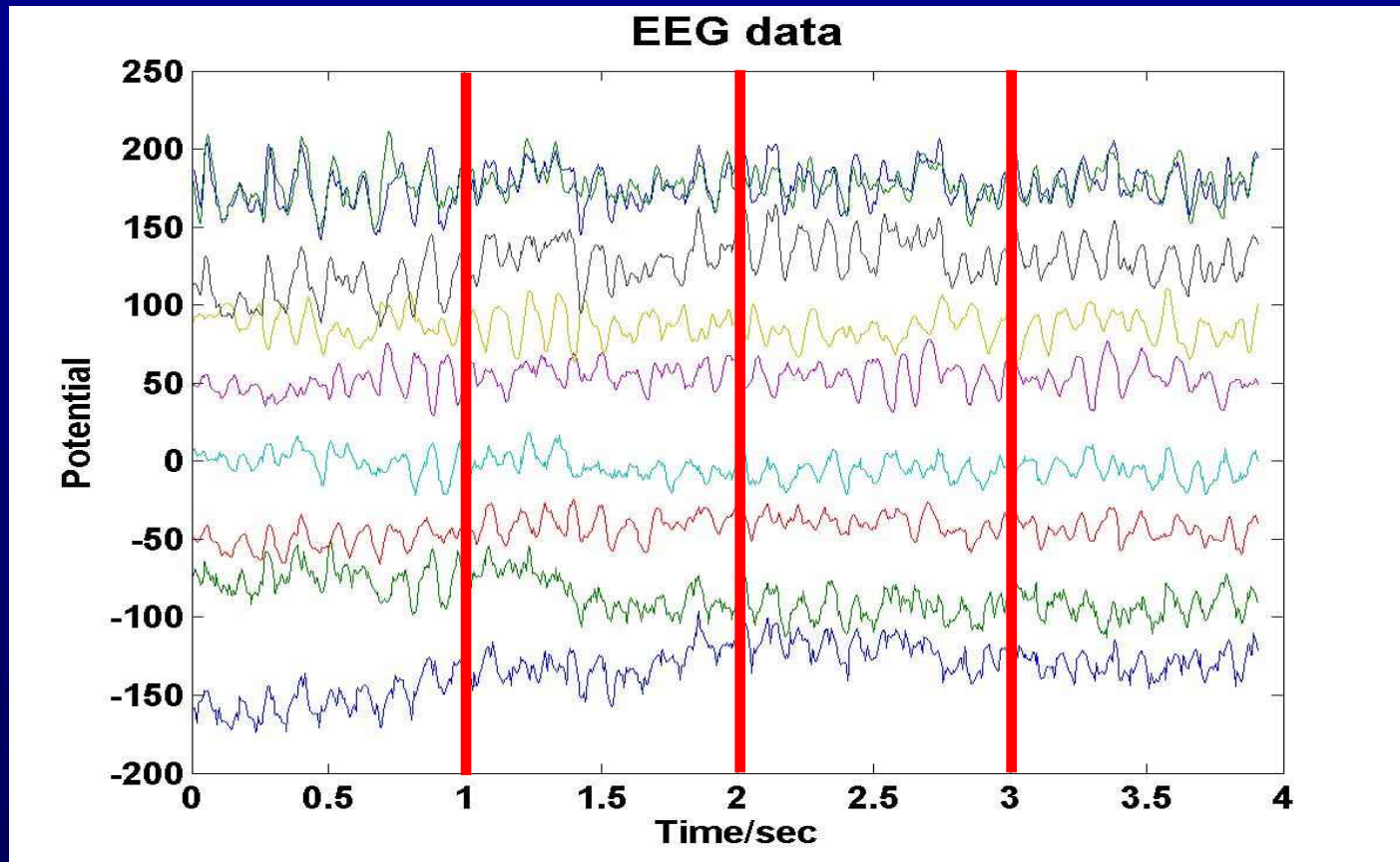
> 10 minutes measurement

> 100 samples/sec

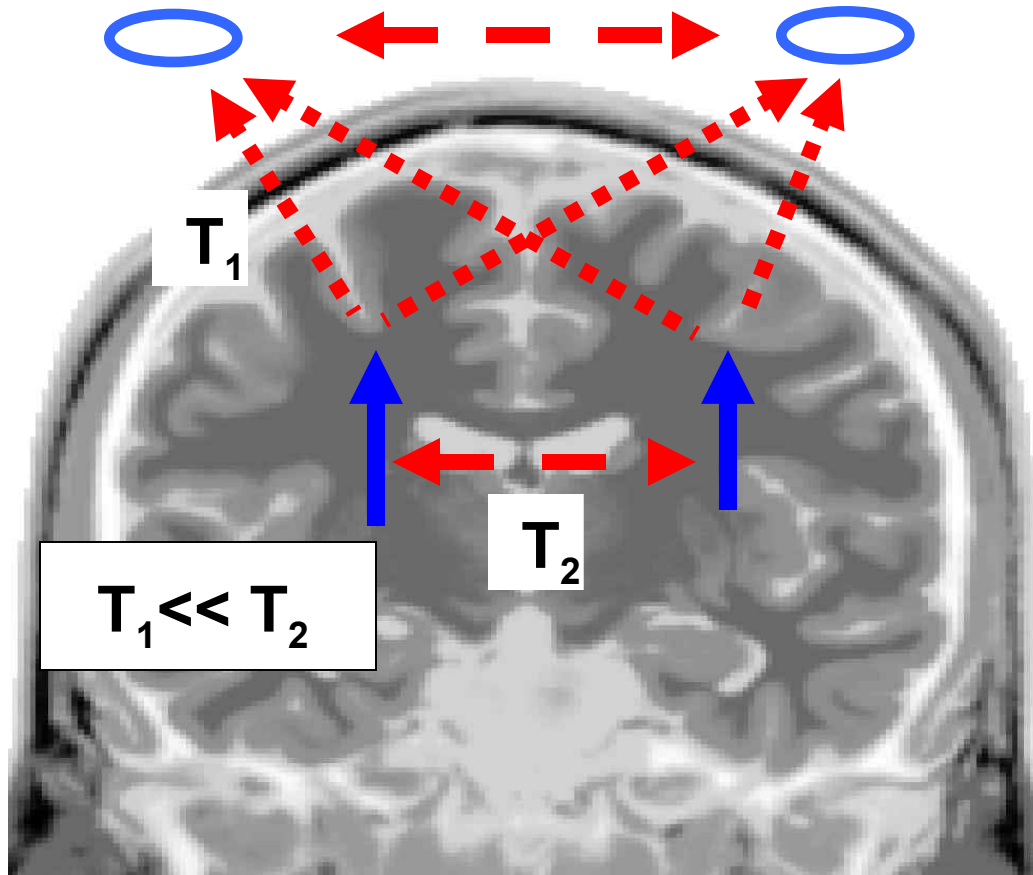
19-128 channels

⇒ Some millions of data points

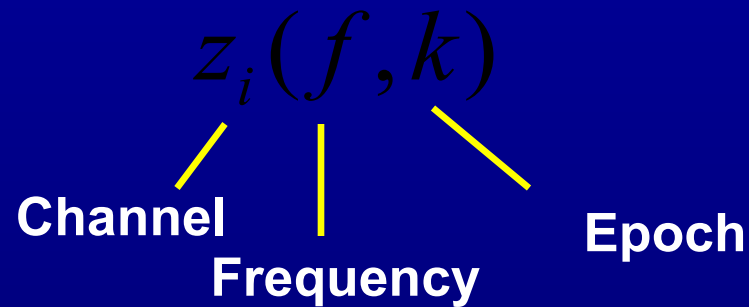
1. Divide data into epochs (e.g. of 1 sec)
2. Make an analysis in each epoch and average over epochs



The Problem of volume conduction



Data:



Cross-spectrum:

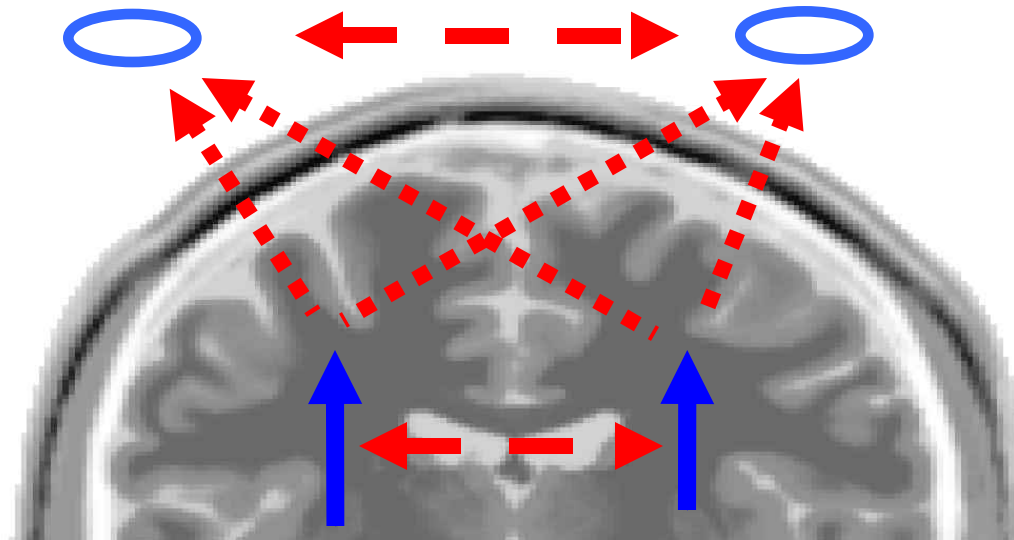
$$S_{ij}(f) = \frac{1}{K} \sum_{k=1}^K z_i(f, k) z_j^*(f, k)$$

Coherency:

$$C_{ij}(f) = \frac{S_{ij}(f)}{\sqrt{S_{ii}(f) S_{jj}(f)}}$$

Independent sources do not contribute to the imaginary part of the cross-spectrum

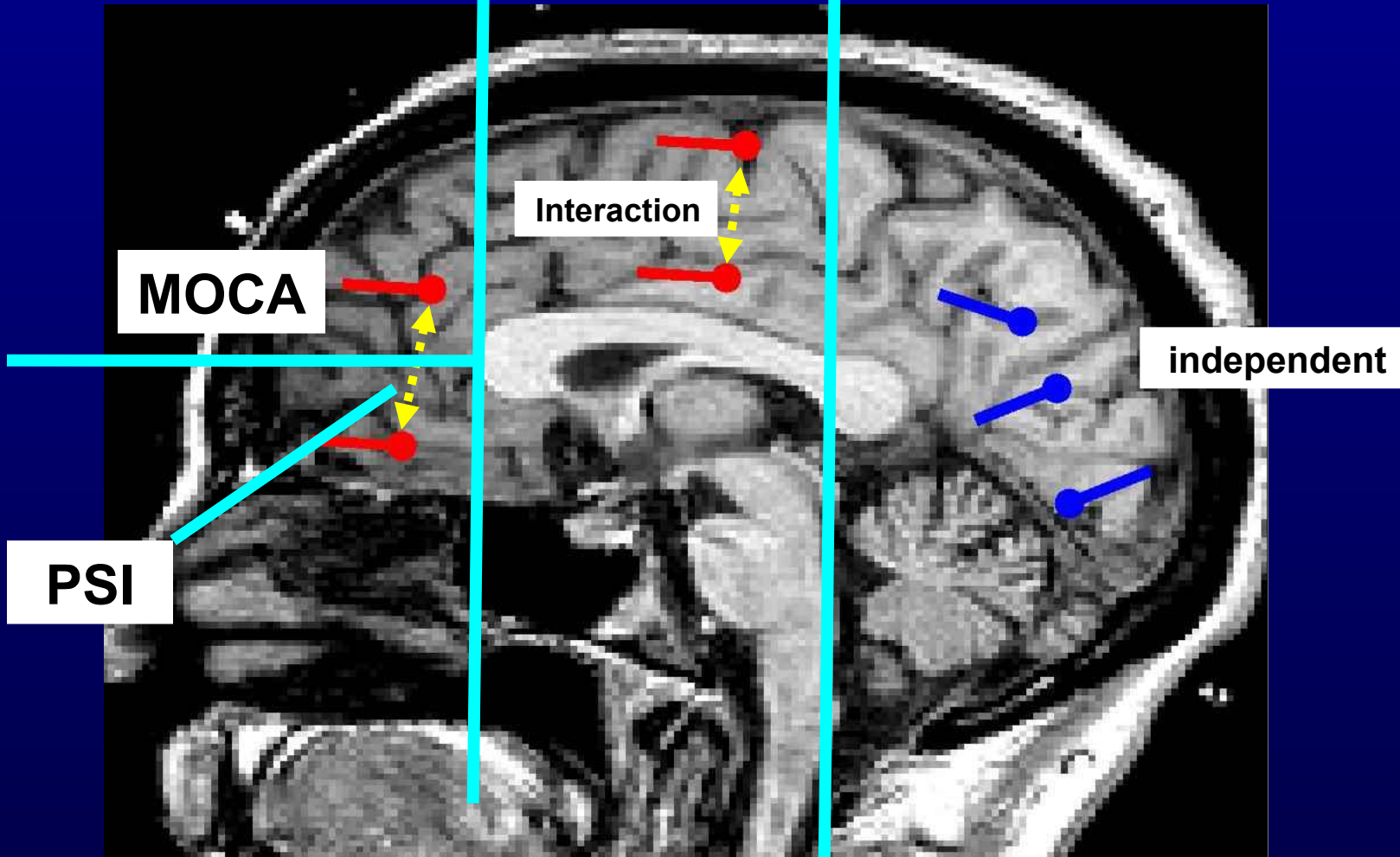
$$S_{12}(f) = \text{Re}(S_{12}(f)) + i \text{Im}(S_{12}(f))$$



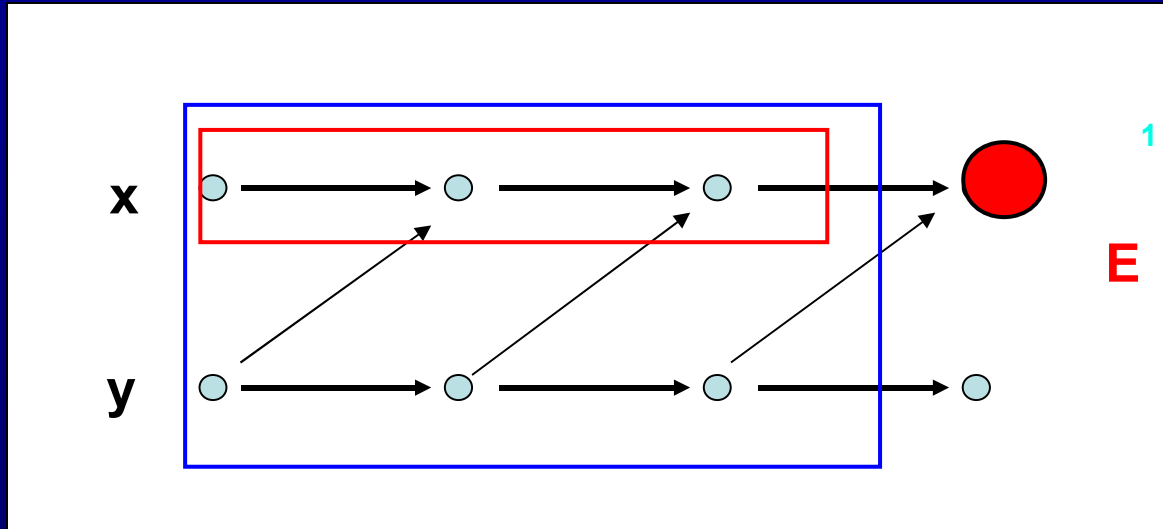
$$S = L \begin{pmatrix} P_1 & 0 & \dots & 0 \\ 0 & P_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & P_M \end{pmatrix} L^T$$

**PISA
sPCA**

Imaginary parts



Granger causality



$$F_{y \rightarrow x} = \log \left(\frac{E_1}{E_2} \right)$$

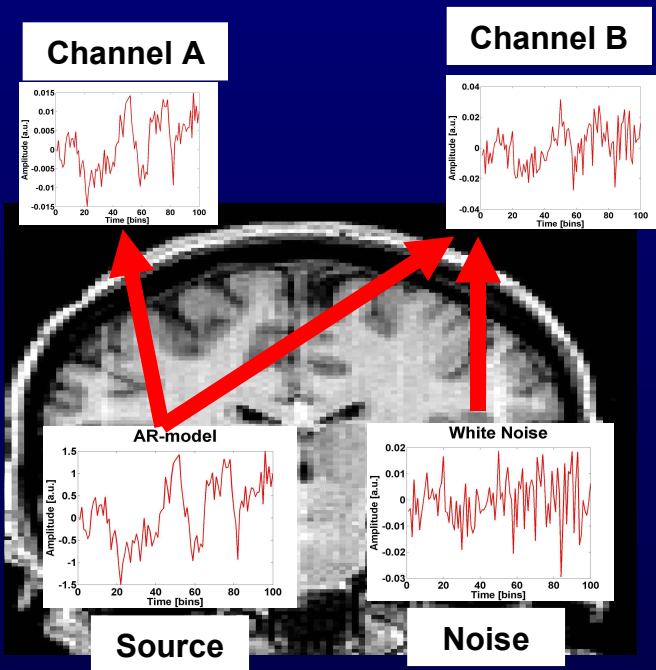
$$\hat{G} = F_{x \rightarrow y} - F_{y \rightarrow x}$$

$$G = \frac{\hat{G}}{std(\hat{G})}$$

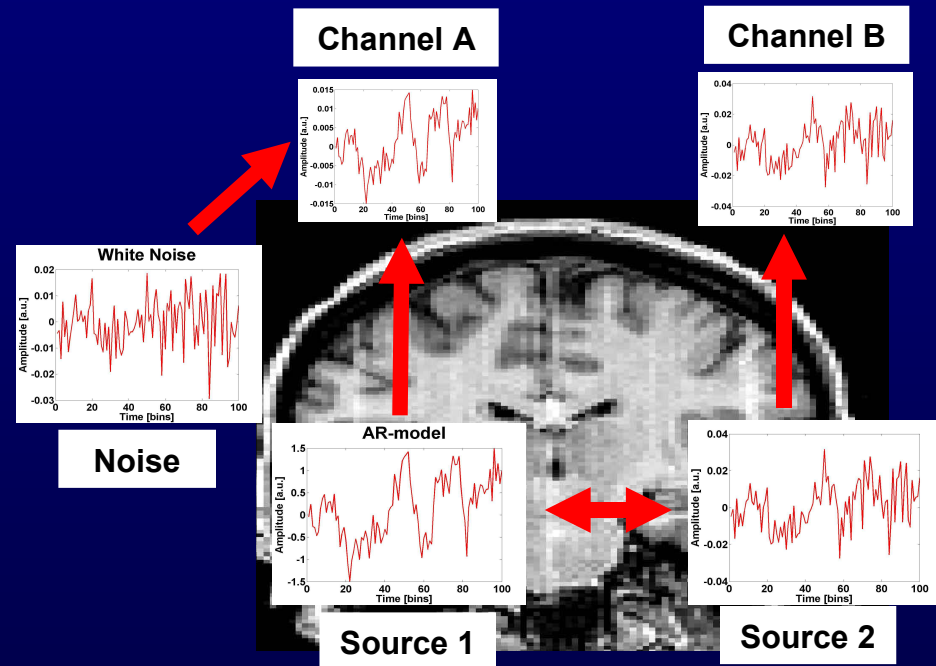
Motivation

- Many measurements like EEG/MEG/fMRI are extremely noisy

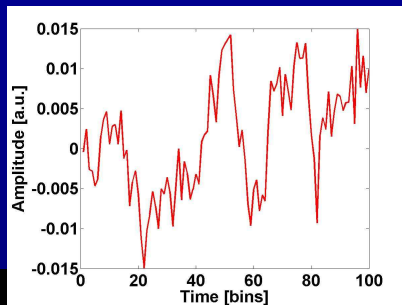
Mixtures of independent sources:
Do we estimate fake direction?



Additive noise:
Do we estimate wrong direction?



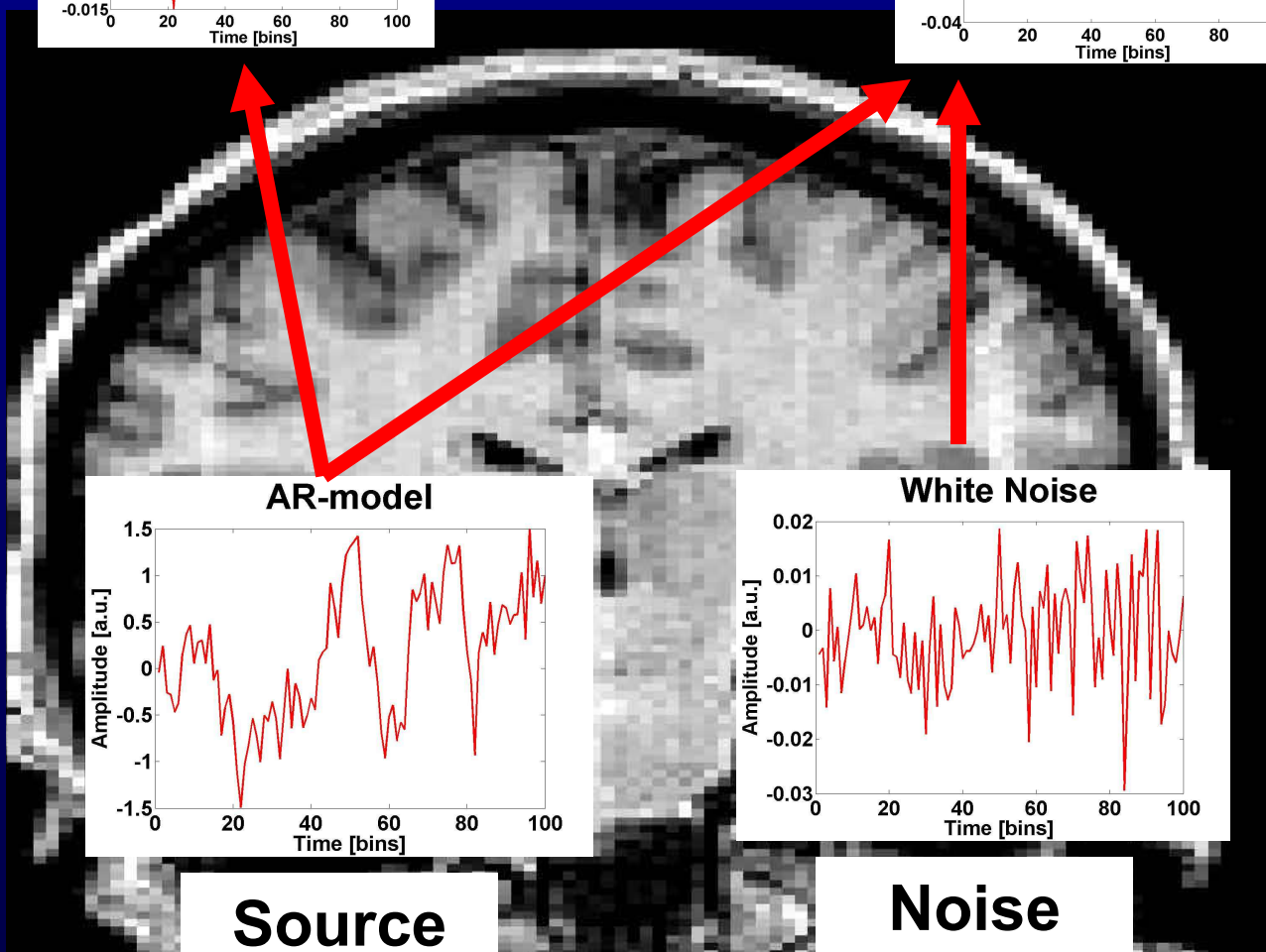
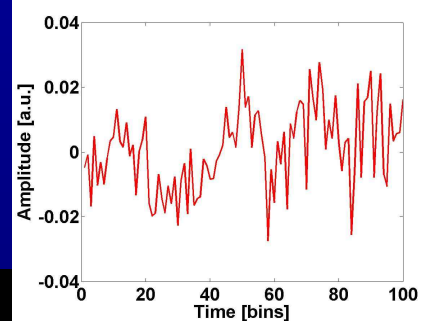
Channel A



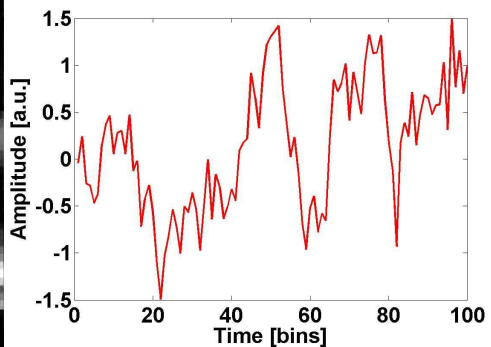
**Granger
Causality**



Channel B

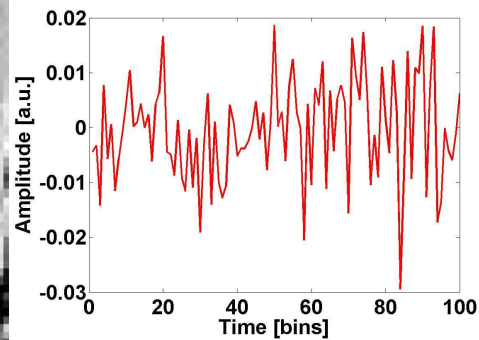


AR-model



Source

White Noise



Noise

Phase Slope Index (PSI)(ψ)

Observations:

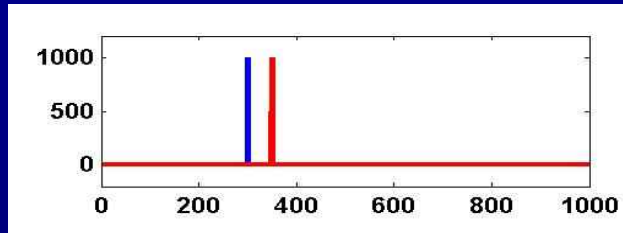
A Independent sources do not contribute to the imaginary part of the cross-spectrum

B Slope of phase of cross-spectrum indicates direction

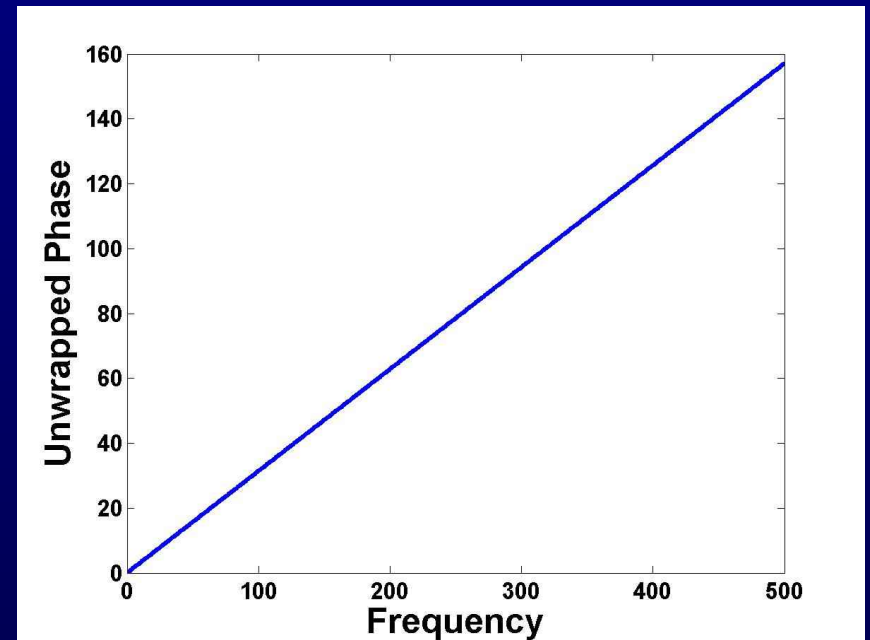
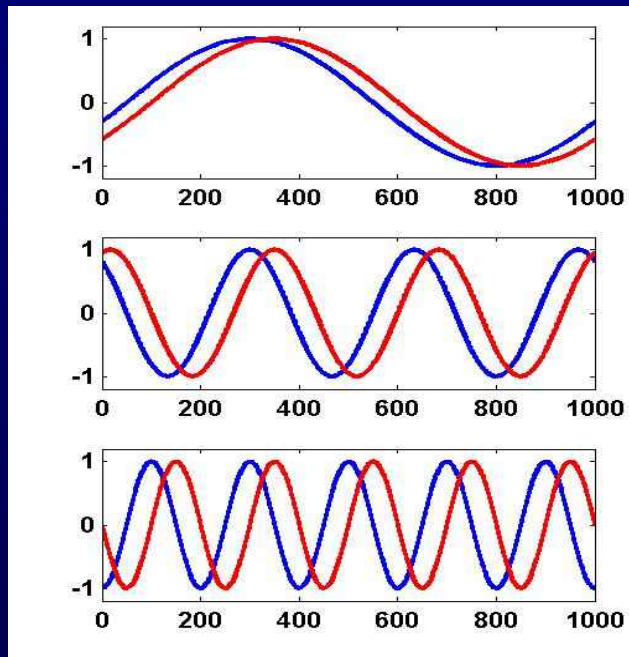
B

Slope of phase-spectrum indicates temporal ordering

Data

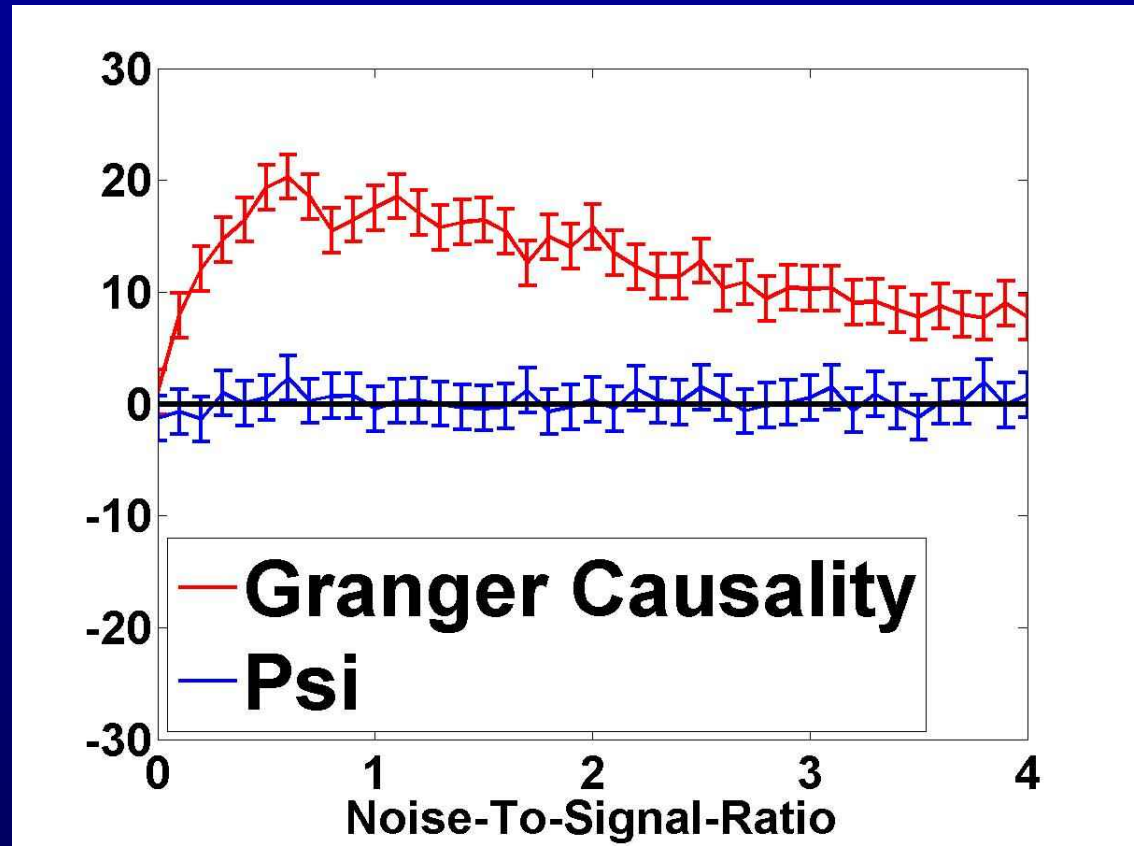
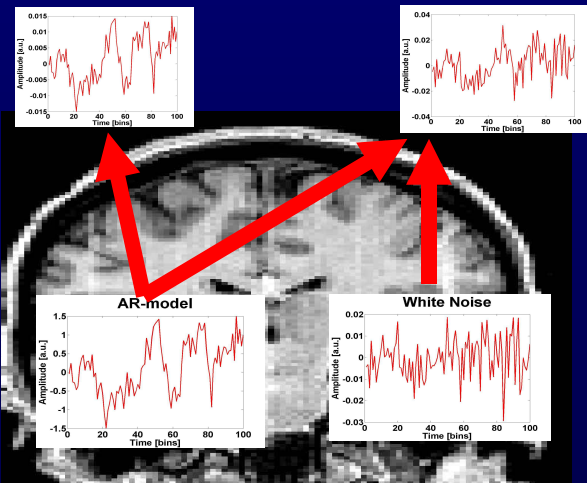


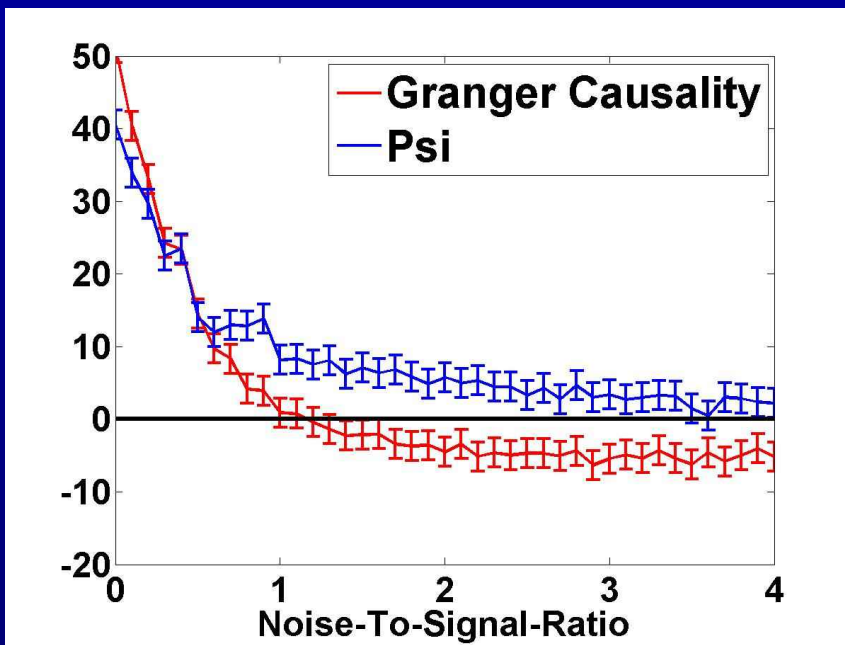
Decomposition



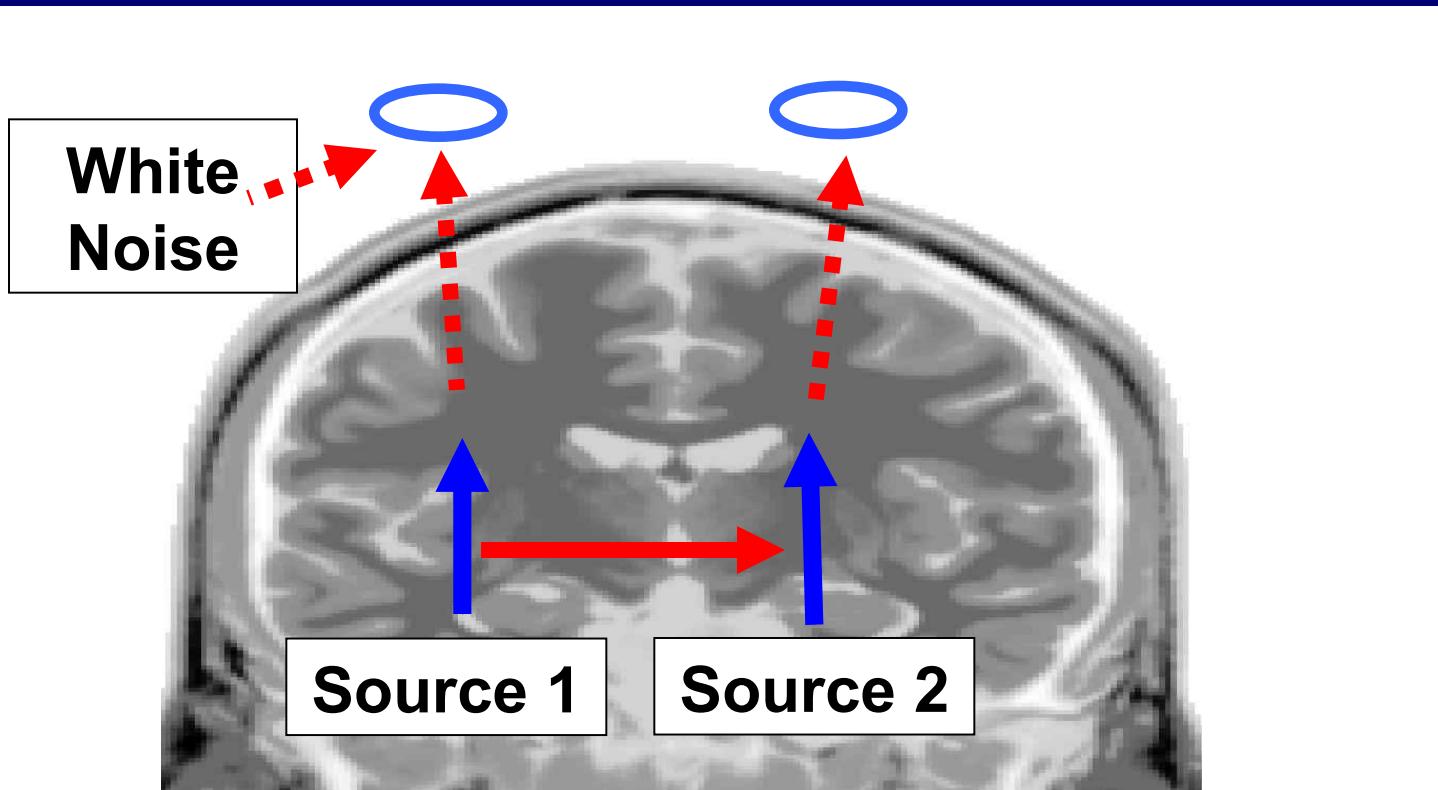
Combining A and B: Average of Phase-Slope such that it is insensitive to mixtures

$$\begin{aligned} & \frac{1}{N} \sum_f \frac{\Phi(f + \delta f) - \Phi(f)}{\delta f} \\ & \approx \frac{1}{N} \sum_f \frac{\sin(\Phi(f + \delta f) - \Phi(f))}{\delta f} \\ & \longrightarrow \sum_f |C(f + \delta f)C(f)| \sin(\Phi(f + \delta f) - \Phi(f)) \\ & = \text{Im} \left(\sum_f C(f + \delta f)C^*(f) \right) \equiv \Psi \end{aligned}$$

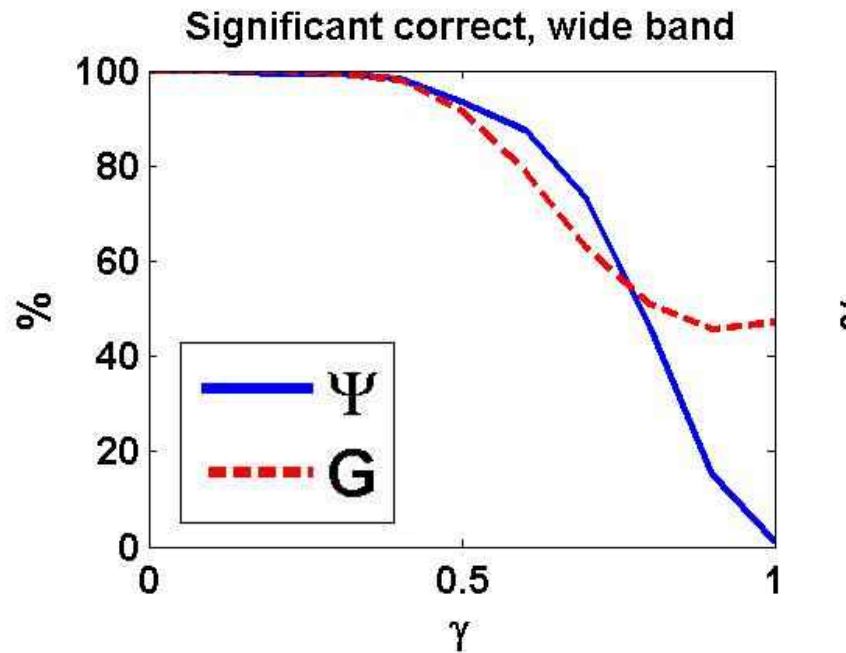




$$A(1) = \begin{pmatrix} .95 & 0 \\ .95 & .5 \end{pmatrix}$$



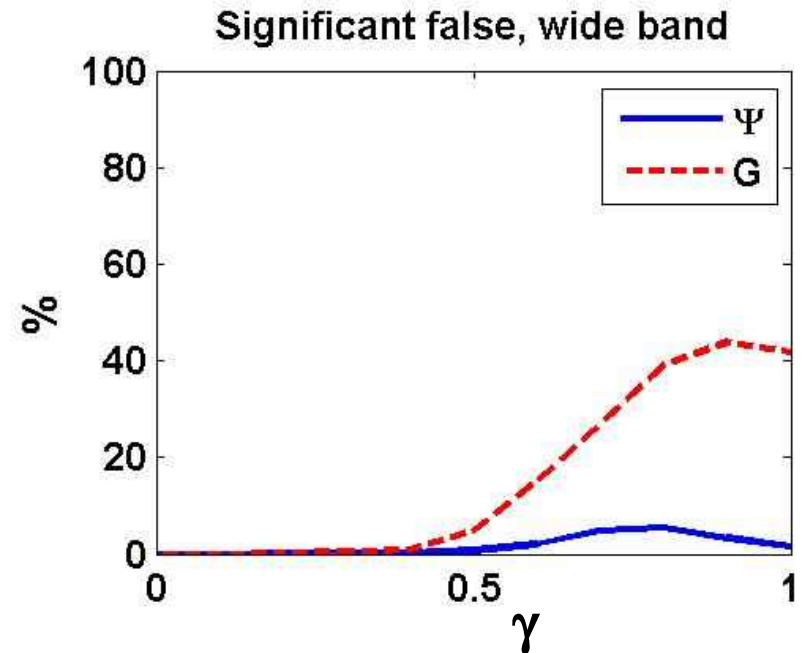
Simulation: $(1-\gamma)$ true flow + γ mixed noise Comparison with Granger Causality



No
Noise



Only
Noise

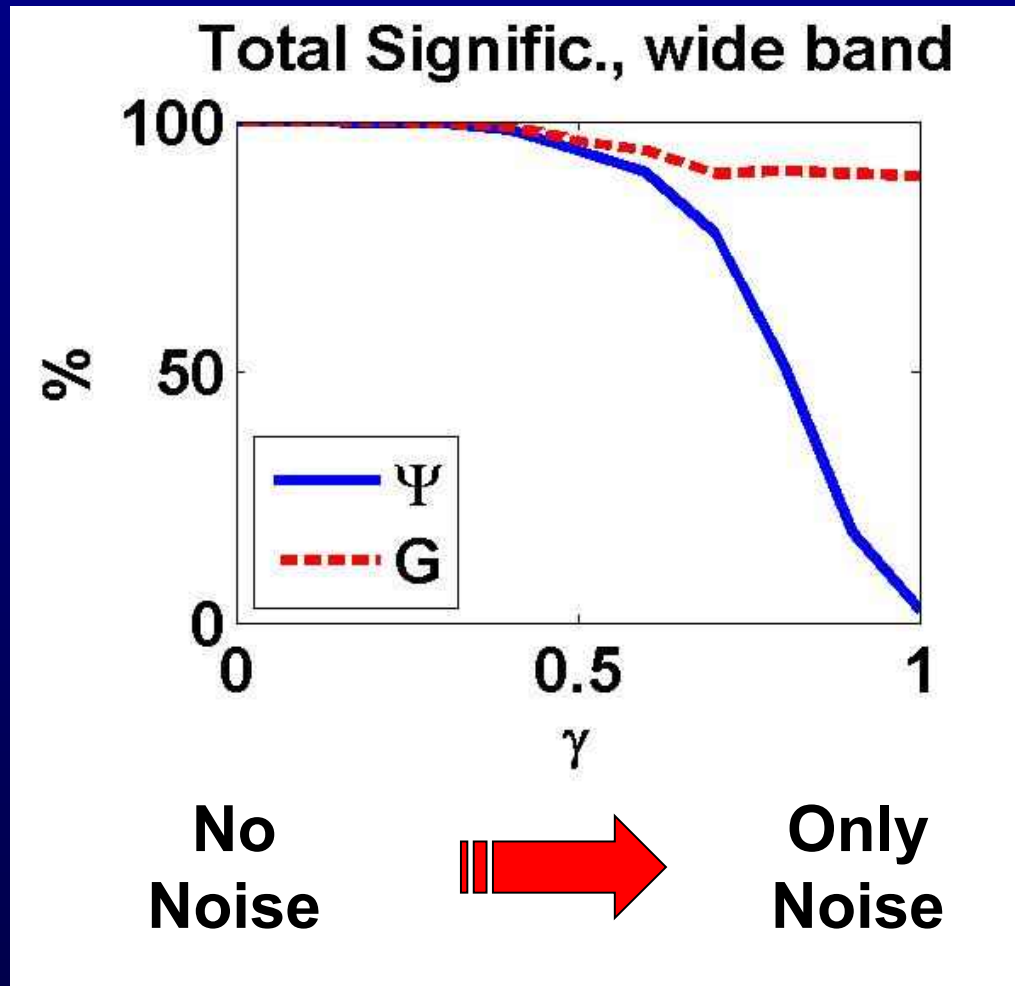


No
Noise



Only
Noise

“I know that I don’t know anything”
Sokrates



Challenge

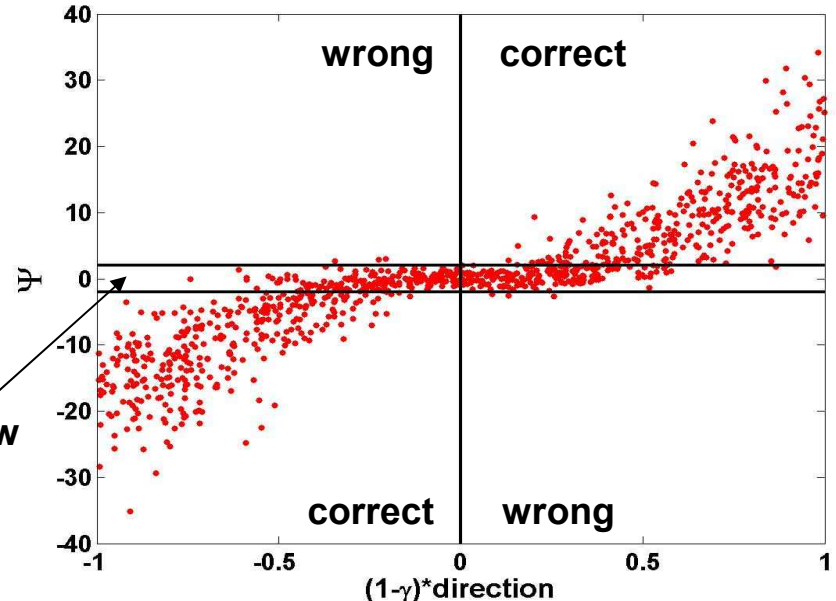
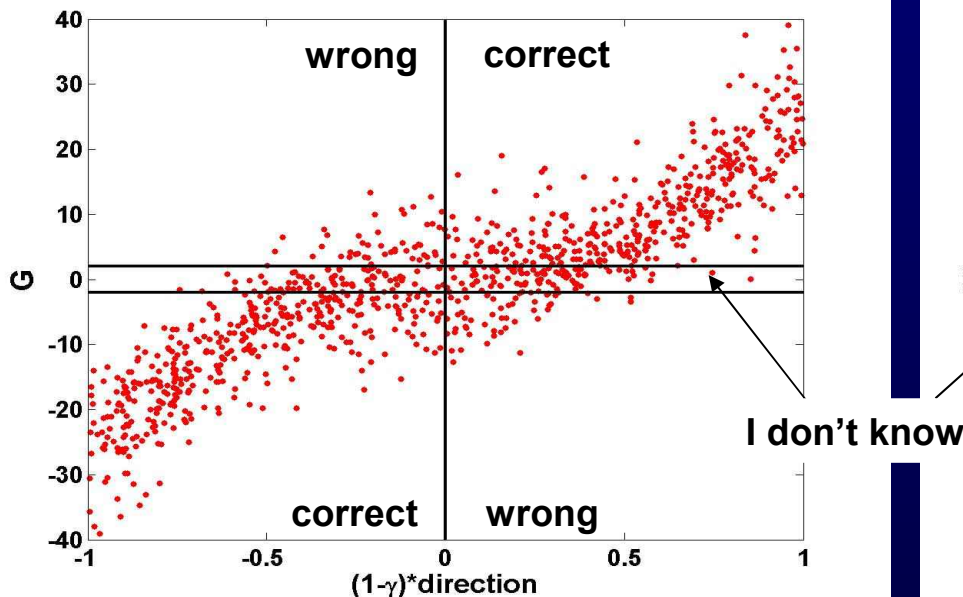
- **correct:** **+1 point**
- **wrong:** **-10 points**
- **“I don ‘t know”:** **0 points**

Granger causality

Correct	wrong	Total points
736	100	-264

Phase Slope Index

Correct	wrong	Total points
638	6	578



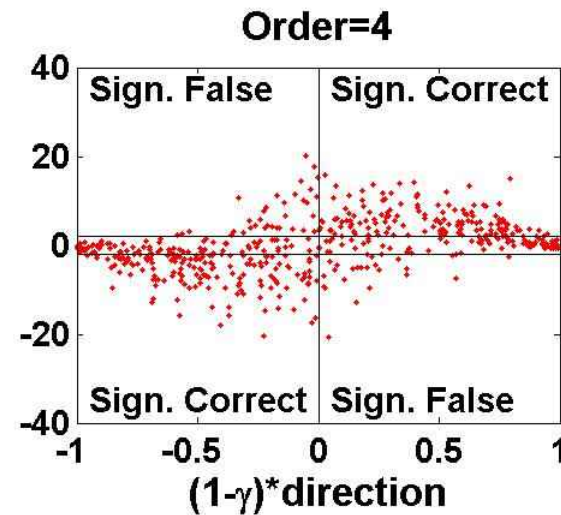
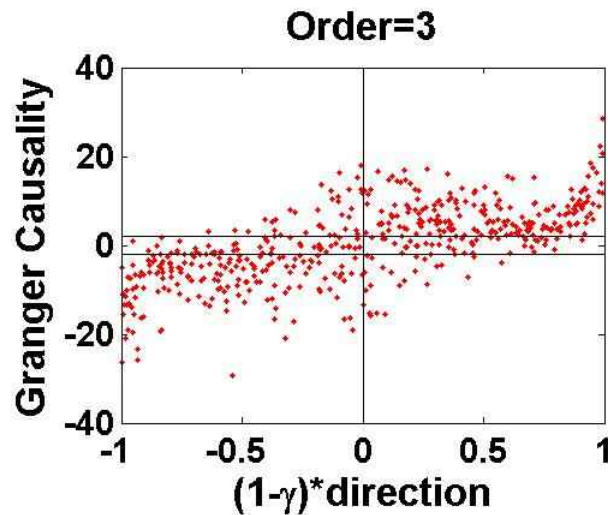
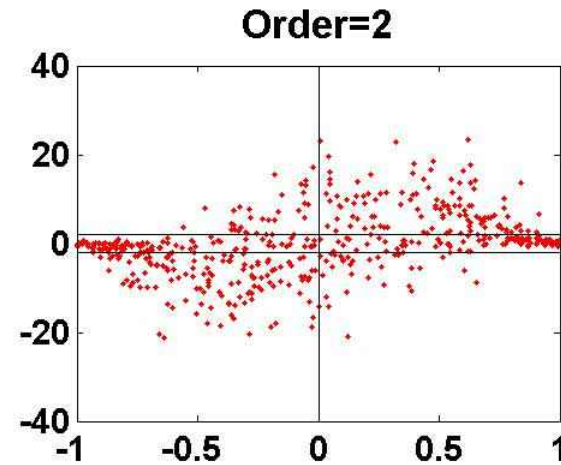
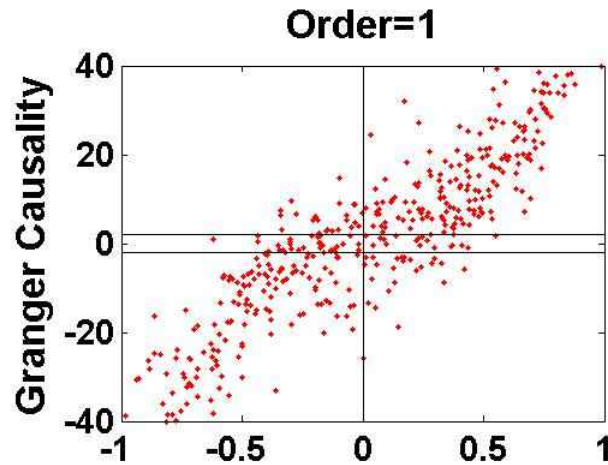
What matters:

Simulated challenge data:

- problem is generic (details are open to discussion)
- evidence is weighted

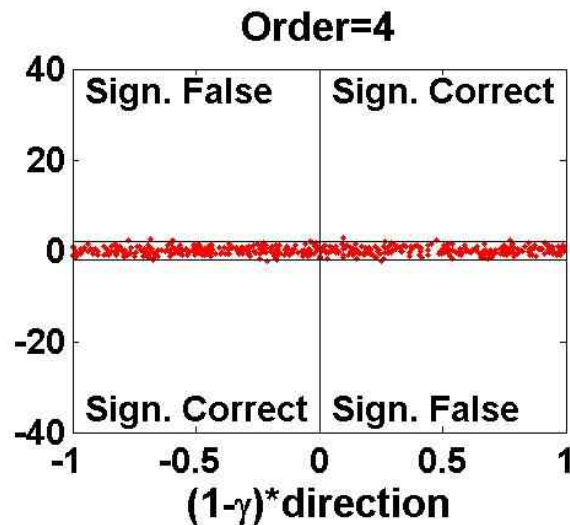
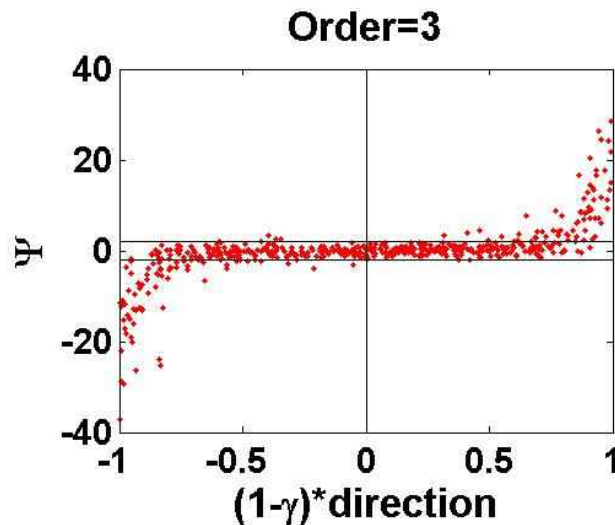
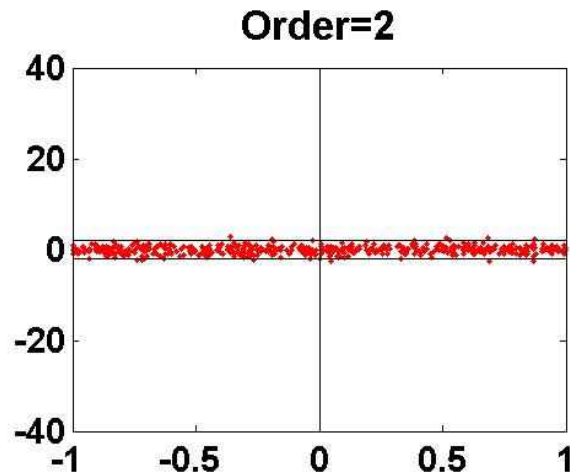
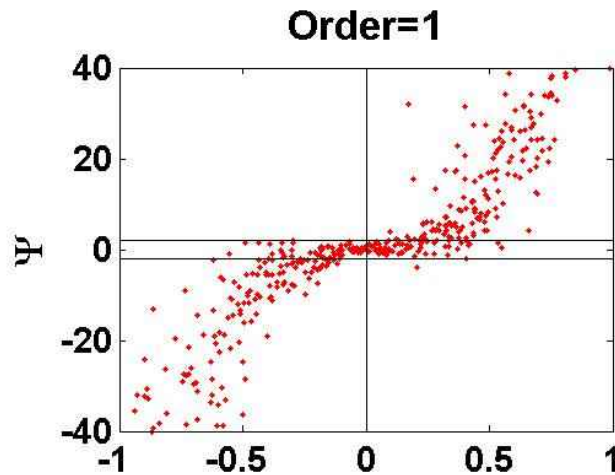
Nonlinearity of order k

Granger Causality



Nonlinearity of order k

PSI



FAQ

Question

Answer

Nonlinear systems?

**alright with exceptions
In general; it possible but difficult to
construct counterexamples**

Direct vs. indirect ?

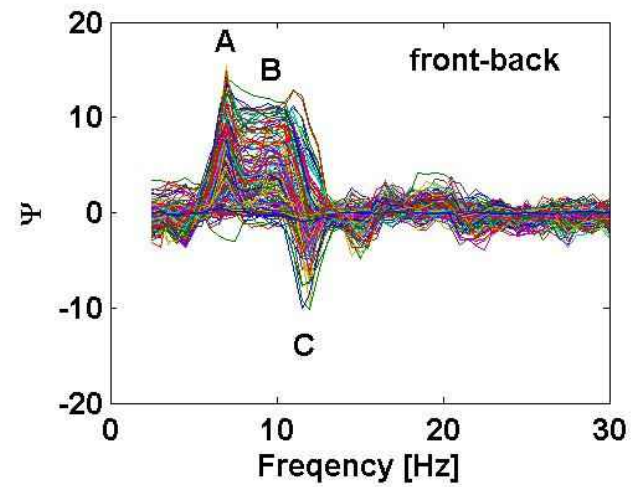
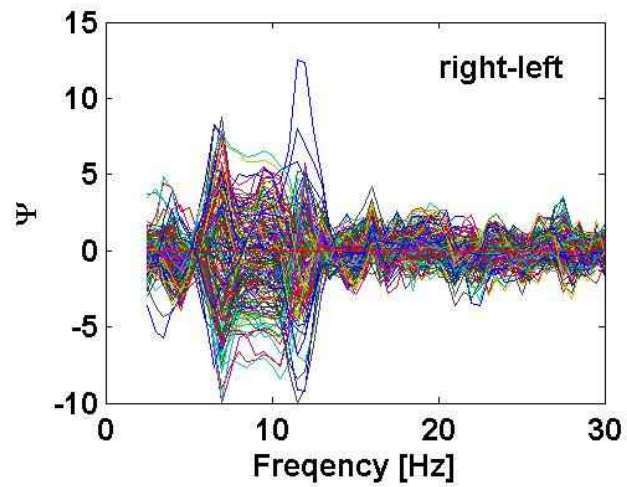
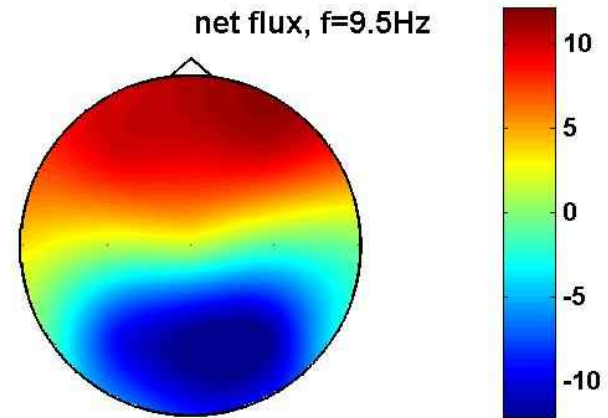
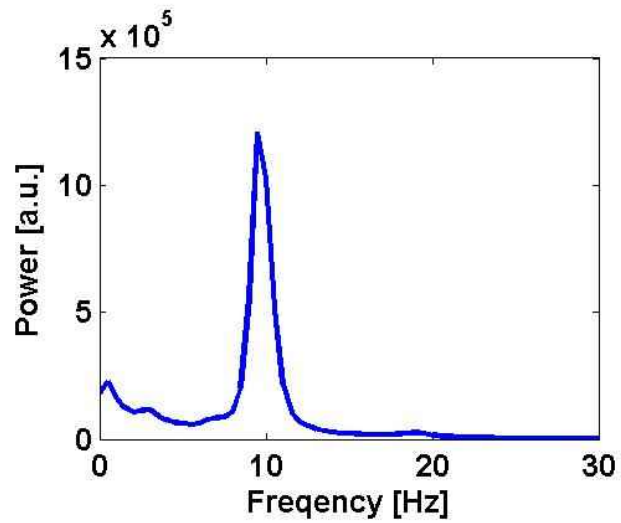
partialing (rarely) possible

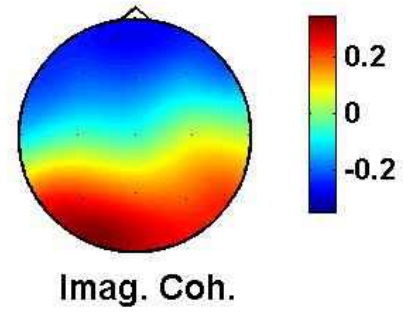
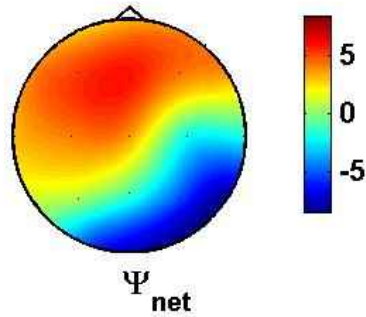
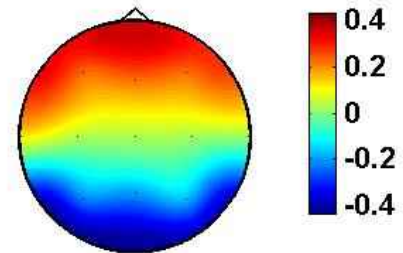
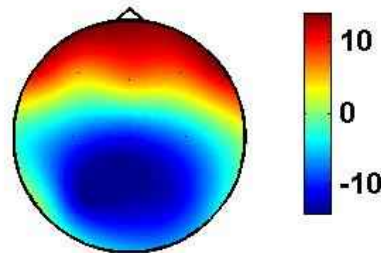
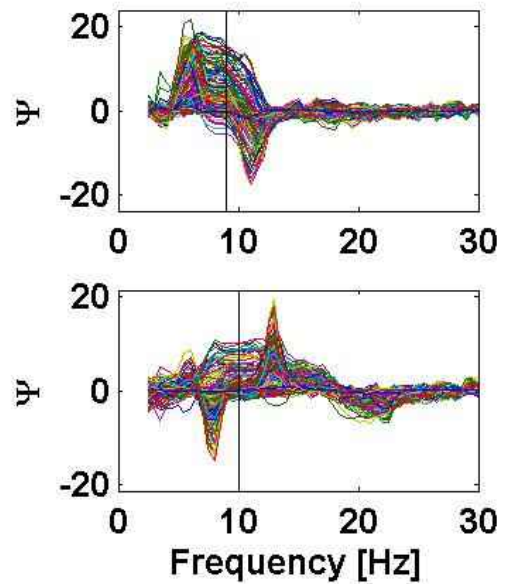
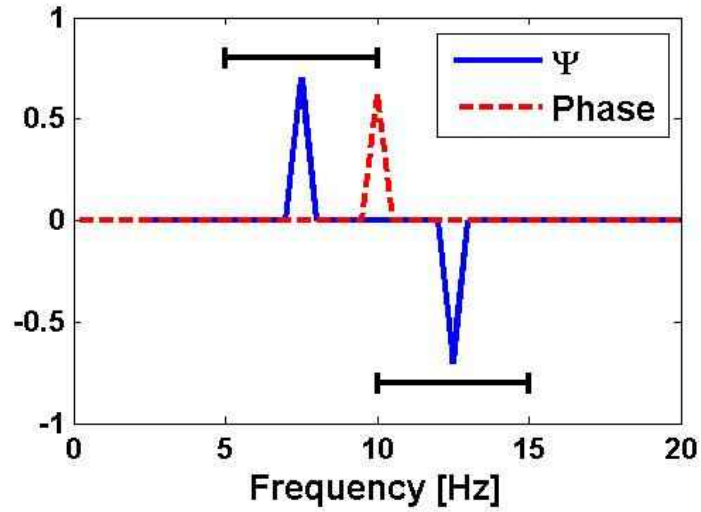
Bidirectional flux?

**PSI is (trivially) correct,
(impossible to resolve completely)**

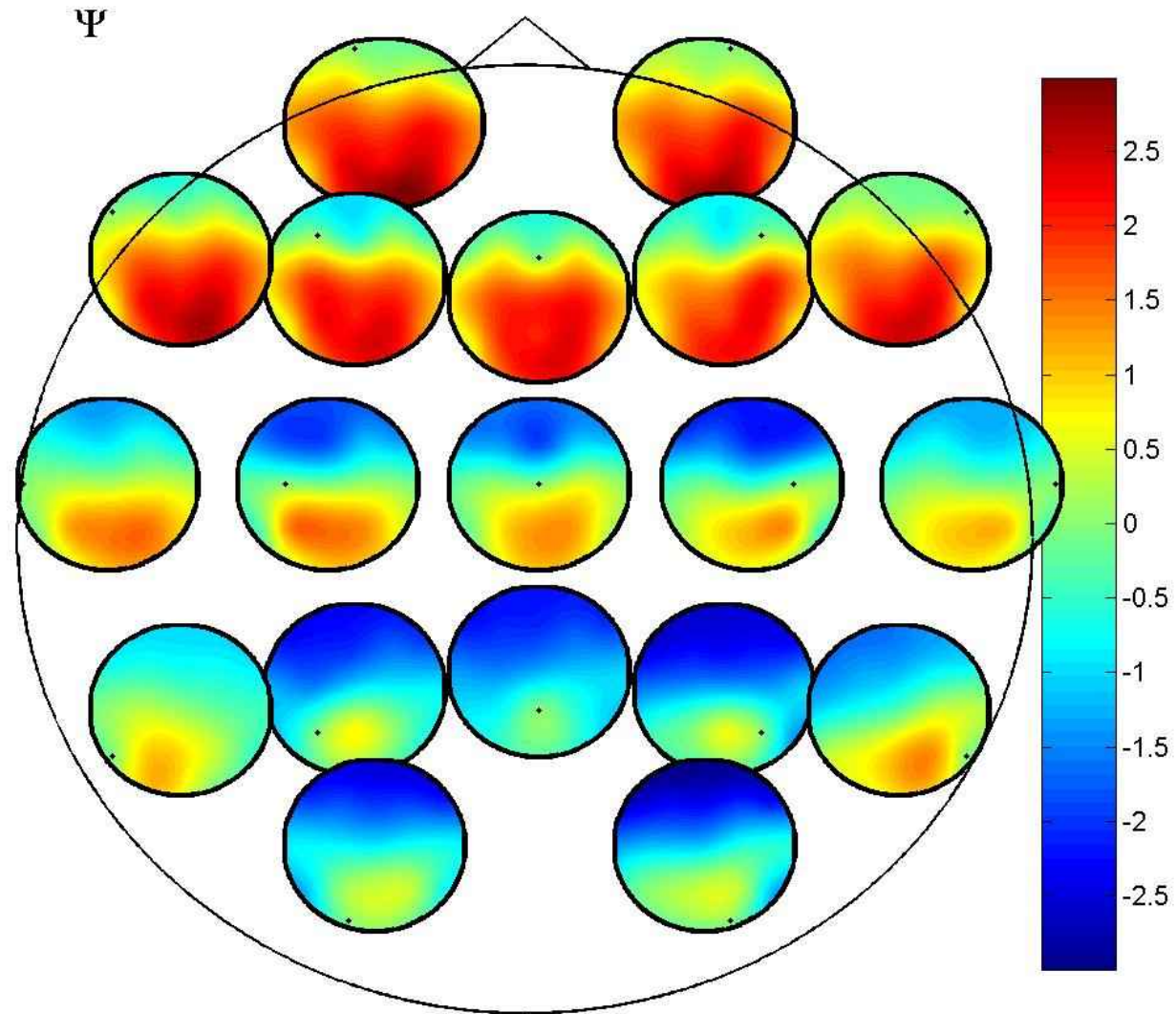
Estimate delay?

**Not in the presence of noise,
Results are really binary**





Alpha rhythm, Eyes closed, 88 subjects



Surrogate Data to test for artefacts of volume conduction

$$\text{Data } \vec{x}(t) = (x_1(t), \dots, x_n(t))$$

1. Demix with ICA

$$\vec{s}(t) = W\vec{x}(t)$$

2. Delay i.th component
by $(i-1) * T$

$$v_1(t) = s_1(t)$$

$$v_2(t) = s_2(t + T)$$

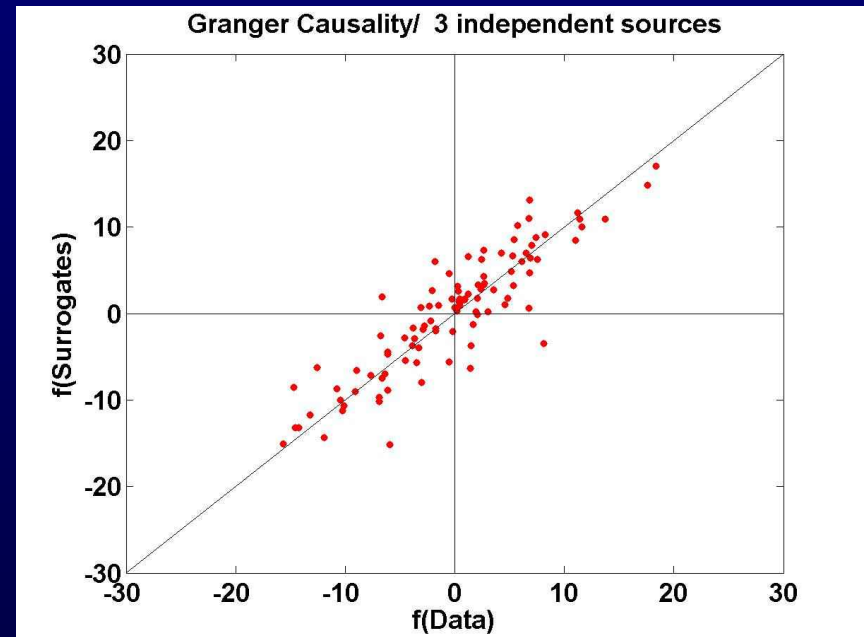
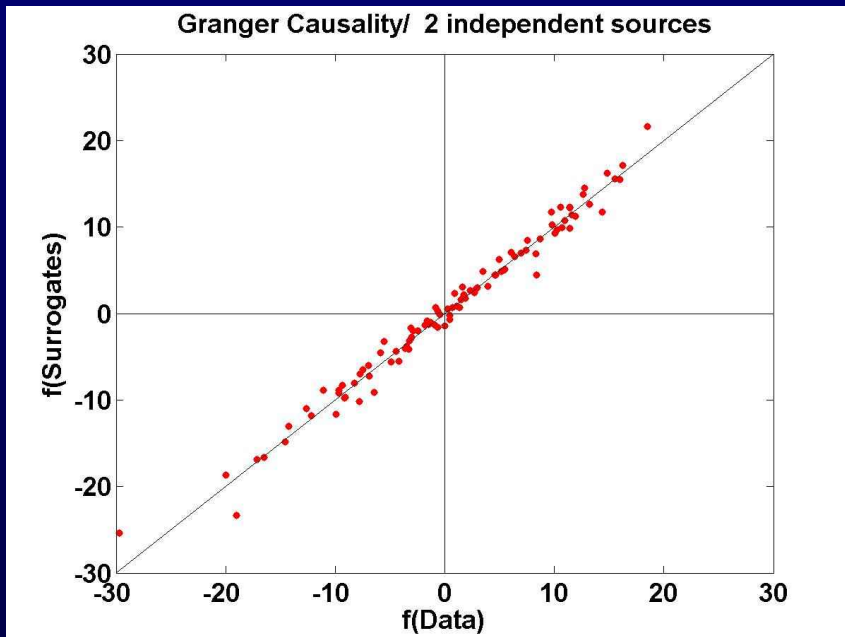
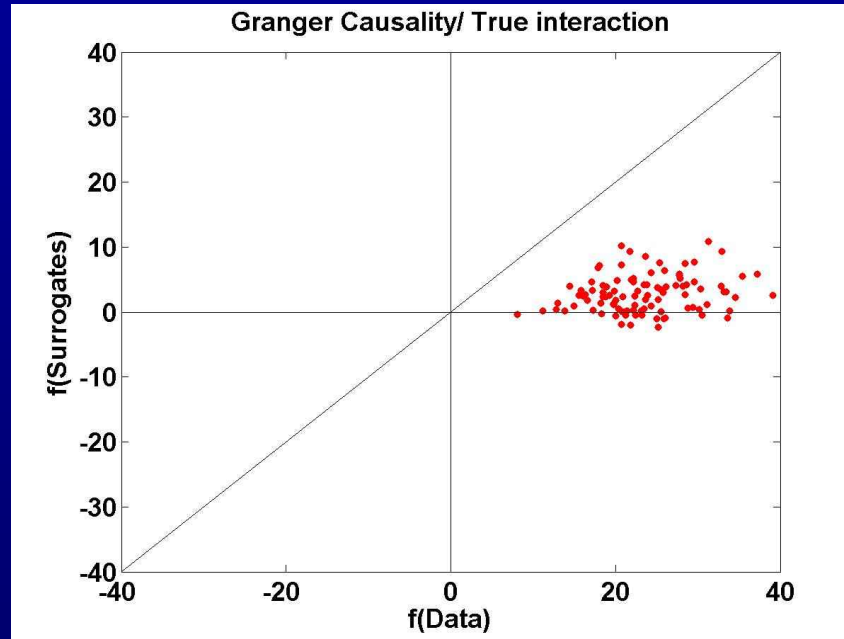
$$v_3(t) = s_3(t + 2T)$$

\vdots

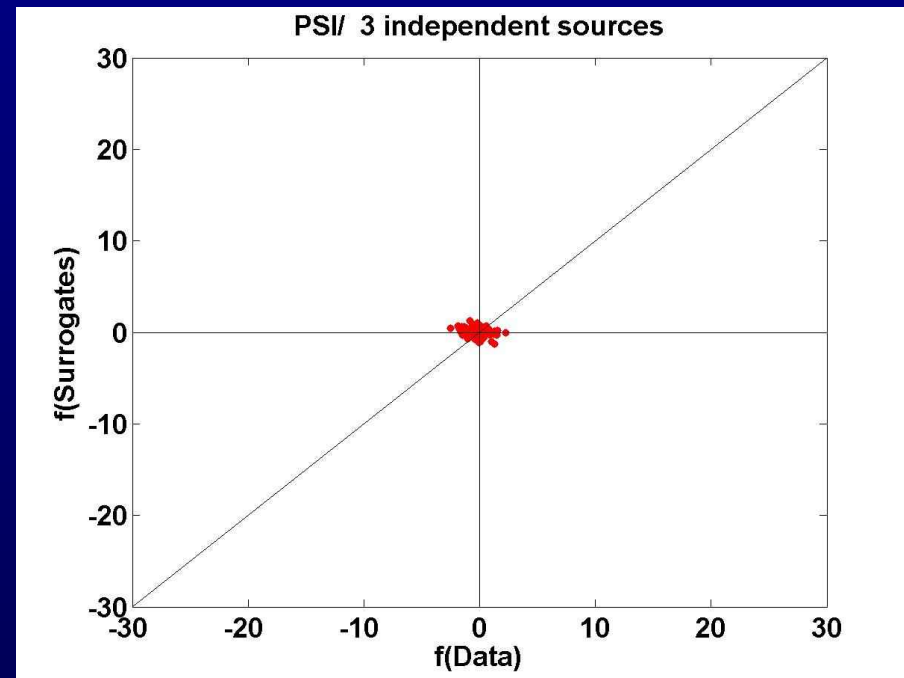
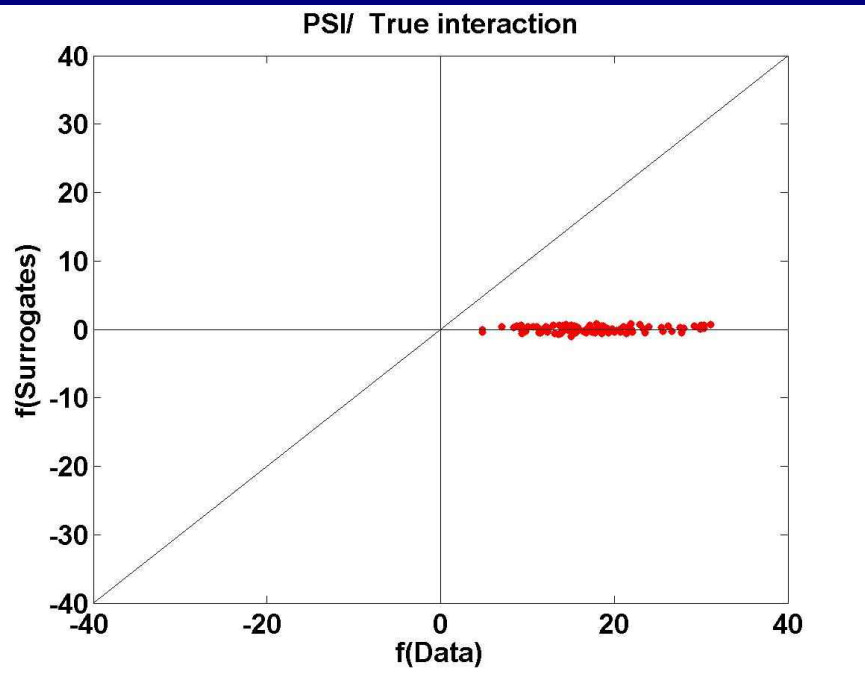
3. Remix

$$\vec{x}_{\text{SURR}}(t) = W^{-1}\vec{v}(t)$$

Granger Causality, Data vs. Surrogates



Phase Slope Index, Data vs. Surrogates



Summary

- Imaginary parts of cross-spectra is not affected by non-interacting sources \Rightarrow valuable quantity to study interactions
- Direction with “Phase Slope Index” (PSI)
- Surrogates with ICA

Thanks to

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Forooz Shahbazi