

VIII Rencontre de São Carlos - Singularités réelles et complexes

VIII Encontro de São Carlos - Singularidades reais e complexas

VIII São Carlos Workshop - Real and Complex Singularities

# São Carlos em Marselha

## Abstractos

1. Minicourses .....	P. 2
2. Plenaries .....	P. 6
3. Parallel lectures .....	P. 14
4. Posters. ....	P. 39

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# 1. Mini-courses

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VICTOR GORYUNOV

ELEMENTARY: LAGRANGEAN AND LEGENDRIAN SINGULARITIES

## 1. Symplectic geometry

Definition of a symplectic manifold. Examples:  $\mathbb{R}^{2n}$ , cotangent bundle of a manifold. Liouville form. Symplectomorphisms.

Isotropic and Lagrangian subspaces. Lagrangian Grassmanian.

Darboux theorem. Weinstein's and Givental's theorems.

Lagrangian fibrations and their local equivalence. Lagrangian maps. Caustics.

Hamiltonian vector fields.

## 2. Contact geometry

Definition of a contact manifold. Examples: projectivised cotangent bundle, 1-jet space of functions on a manifold. Contact Darboux theorem.

Symplectisation. Contact Hamiltonian vector fields. Contactisation.

Legendrian submanifolds. Legendrian fibrations. Legendrian maps. Wave fronts. Legendre transforms.

## 3. Generating families

Symplectic reduction. Generating families of Lagrangian submanifolds, their existence, uniqueness and equivalence.

Generating families of Legendrian submanifolds, their equivalence.

Examples. Wave front propagation, generating functions of canonical transformations.

## 4. Invariants of Legendrian knots

Arnold's conjecture on a necessity of four cusps in a generic eversion of a planar circular front.

Representing Legendrian knots in the standard contact 3-spaces by their wave fronts. Perestroikas of generic planar wave fronts.  $J^+$ -type invariants of fronts. Bennequin number of a Legendrian knot.

Kauffman polynomial of a front. Bennequin number estimate.

Finite order invariants.

Chekanov-Pushkar combinatorial invariant.

### Literature:

1. V. I. Arnold, *Singularities of caustics and wave fronts*, Kluwer, 1990.

2. V. I. Arnold, *Mathematical methods of classical mechanics*.
3. V. I. Arnold, S. M. Gusein-Zade and A. N. Varchenko, *Singularities of differentiable mappings*, vol.1, ch.3.
4. V. V. Goryunov, *Plane curves, wavefronts and Legendrian knots*, Phil. Trans. R. Soc. Lond. A (2001) **359**, 1497–1510.

MAXIM KAZARIAN

ADVANCED: COMPUTATION OF THOM POLYNOMIALS

I. DERIVED PORTEOUS-THOM CLASSES

II. LAGRANGE AND SYMMETRIC DEGENERACIES

*Note of organizers: Here is the beginning of the notes of the course of Maxim Kazarian. The complete notes will be distributed at the beginning of the week.*

**Notational remarks.** 1. We often use the same notation  $E$  for a vector bundle  $E \rightarrow M$  and for the restriction of this bundle to a submanifold  $X \subset M$ . Moreover, we keep the notation  $E$  for the pull-back  $\pi^*E$  for any smooth map (not necessary locally trivial fibration)  $\pi : X \rightarrow M$ . The same agreement holds for characteristic classes of vector bundles. So we write often  $c_i(E) \in H^*(X)$  instead of  $c_i(\pi^*E)$  or  $\pi^*c_i(E)$ . This agrees, for example, with the fact that the push-forward homomorphism  $\pi_* : H^*(X) \rightarrow H^*(M)$  is a homomorphism of  $H^*(M)$ -modules: in our notation the projection formula claims:

$$\pi_*(c_i(E) \alpha) = c_i(E) \pi_*(\alpha) \in H^*(M), \quad \alpha \in H^*(X).$$

The experience shows that this agreement does not lead to ambiguity but simplifies considerably notations. If we need, nevertheless, to indicate explicitly the base of the bundle then we prefer to denote it by  $E|_X$  rather than by  $\pi^*E$ .

2. All results of this paper and their proofs hold in the situation of nonsingular algebraic varieties over any algebraically closed ground field and Chow rings instead of cohomology: the groups  $H^{2k}(M)$  can be replaced in all formulas by  $A_{m-k}M$  without any change, where  $m = \dim M$ .

## Derived Porteous-Thom classes

We study singularity loci of a holomorphic map  $f : M \rightarrow N$ . Porteous-Thom singularities  $\Sigma^r$  are determined by the 1-jet. Therefore, it is more natural

to consider these singularities in the more general context of a morphism  $\varphi : A \rightarrow B$  of complex vector bundles over some smooth base  $M$ . The case of a map is reduced to this one by setting  $A = TM$ ,  $B = f^*TN$ ,  $\varphi = f_*$ .

**Definition.** A vector bundle map  $\varphi : A \rightarrow B$  has *Thom-Porteous singularity*  $\Sigma^r$  at a point  $x \in M$  if  $\varphi$  has the kernel rank at least  $r$  at this point, i.e. if  $\dim \ker(\varphi_x : A_x \rightarrow B_x) \geq r$ .

The locus  $\Sigma^r = \Sigma^r(\varphi) \subset M$  is formed by the points with this singularity,

$$\Sigma^r(\varphi) = \{x \in M \mid \dim \ker \varphi_x \geq r\}.$$

In what follows we assume that the morphism  $\varphi$  considered as a section of the bundle  $\text{Hom}(A, B)$  is *generic*, i.e. it is transversal to the locus  $\Sigma^r$  of this singularity in the total space of the bundle  $\text{Hom}(A, B)$  for all  $r$ . In this case  $\Sigma^r(\varphi)$  is a reduced subvariety in  $M$  of the ‘expected’ codimension

$$\text{codim } \Sigma^r = r(r + \ell), \quad \ell = \text{rk } B - \text{rk } A.$$

In any case the constructions of this section can be applied directly to the total space of the bundle  $\text{Hom}(A, B)$  instead of the base  $M$  and to the locus  $\Sigma^r$  in this space (which does not depend on the choice of a particular section).

In general the locus  $\Sigma^r$  is not smooth: its smooth part  $\overset{\circ}{\Sigma}^r = \Sigma^r \setminus \Sigma^{r+1}$  is formed by the points with the kernel rank equal exactly to  $r$ . The first step in the study of the singularity  $\Sigma^r$  consists in resolving this singularity.

**Definition.** The *standard resolution of singularities* of the locus  $\Sigma^r$  is defined as the natural projection

$$p : \tilde{\Sigma}^r \rightarrow M$$

of the space  $\tilde{\Sigma}^r = \tilde{\Sigma}^r(\varphi)$  formed by all couples of the form  $(x, K_x)$  where  $x$  is a point of the base  $M$ , and  $K_x$  is an  $r$ -dimensional subspace of  $A_x$  contained in the kernel  $\ker \varphi_x$ .

It is clear that  $p(\tilde{\Sigma}^r) = \Sigma^r$  and it is one-to-one over  $\overset{\circ}{\Sigma}^r \subset \Sigma^r$ . The set  $\tilde{\Sigma}^r$  can be considered as a subspace of the associated Grassmann bundle  $\pi : G_r(A) \rightarrow M$  given as the zero locus of a section of the bundle  $\text{Hom}(K, \pi^*B)$ , where  $K$  is the tautological vector bundle of rank  $r$ , and where the section is given by the natural morphism  $K \hookrightarrow \pi^*A \xrightarrow{\varphi} \pi^*B$ . If the genericity condition for  $\varphi$  holds then this section is transversal to the zero section, hence,  $\tilde{\Sigma}^r$  is a smooth submanifold in  $G_r(A)$ .

We keep the notation  $K$  for the restriction of that bundle to  $\tilde{\Sigma}^r$ . The restriction of this bundle to the open part  $p^{-1}(\tilde{\Sigma}^r) \simeq \tilde{\Sigma}^r$  coincides with the kernel bundle of the morphism  $\varphi$ . Therefore, we call  $K \rightarrow \tilde{\Sigma}^r$  the *virtual kernel bundle*. Characteristic classes of this bundle are cohomology classes on  $\tilde{\Sigma}^r$ . The following theorem describes the push-forward of these classes to  $M$ .

Recall that the *Schur polynomial*  $\Delta_{\lambda_1, \dots, \lambda_r}(c)$  associated with any sequence of integers  $(\lambda_1, \dots, \lambda_r)$  is the polynomial in variables  $c_1, c_2, \dots$  given by the following  $r \times r$  determinant:

$$\Delta_{\lambda_1, \dots, \lambda_r}(c) = \det \|c_{\lambda_i+j-i}\|_{i,j=1, \dots, r} \begin{vmatrix} c_{\lambda_1} & c_{\lambda_1+1} & \cdots & c_{\lambda_1+r-1} \\ c_{\lambda_2-1} & c_{\lambda_2} & \cdots & c_{\lambda_2+r-2} \\ \vdots & & \ddots & \\ c_{\lambda_r-r+1} & c_{\lambda_r-r+2} & \cdots & c_{\lambda_r} \end{vmatrix},$$

where we set  $c_0 = 1$  and  $c_i = 0$  for  $i < 0$ .

**0.0.1. Theorem.** *Let  $P$  be any polynomial in the Chern classes  $c_1(K), \dots, c_r(K)$ . Then the push-forward class  $p_*P \in H^*(M)$  is given by a universal polynomial (determined uniquely by  $P$  and by  $\ell = \text{rk } B - \text{rk } A$ ) in the relative Chern classes  $c_i(B-A)$ .*

*This polynomial is given by the following explicit formula. According to the splitting principle, set formally  $c(K) = \prod_{i=1}^r (1 - t_i)$ , substitute to  $P$  the corresponding symmetric functions in  $-t_i$  and expand the brackets. Then  $p_*$  is given on the resulting monomials by appropriate Schur polynomials,*

$$p_* t_1^{s_1} \cdots t_r^{s_r} \Delta_{\ell+r+s_1, \dots, \ell+r+s_r}(c(B-A)). \quad (1)$$

In particular, setting  $P = 1$  we get the *Porteous formula* for the Thom polynomial of  $\Sigma^r$ -singularity:

$$[\Sigma^r] = p_*(1) = \Delta_{\ell+r, \dots, \ell+r}(c(B-A)). \quad (2)$$

The polynomials in  $c_i = c_i(B-A)$  represented in the form  $p_*P$  are called *derived Porteous-Thom classes* or *higher Thom polynomials* of  $\Sigma^r$ -singularities.

The proof of Theorem 0.0.1 is given in Sect. ??, ?? below.

## 2. Plenaries

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PAOLO ALUFFI  
FLORIDA STATE UNIVERSITY

CELESTIAL INTEGRATION, CHERN-SCHWARTZ-MACPERSON CLASSES,  
AND STRINGY INVARIANTS

We introduce a formal integral on the system of varieties mapping properly and birationally to a given one, with value in an associated Chow group. Applications include comparisons of Chern numbers of birational varieties, new birational invariants, ‘stringy’ Chern classes, and a ‘celestial’ zeta function specializing to the topological zeta function.

In its simplest manifestation, the integral gives a new expression for Chern-Schwartz-MacPherson classes of possibly singular varieties, placing them into a context in which a ‘change of variable’ formula holds.

The formalism has points of contact with motivic integration.

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RAGNAR-OLAF BUCHWEITZ  
UNIVERSITY OF TORONTO

DISCRIMINANTS AND FREE DIVISORS

Free divisors are hypersurfaces that are reduced, but “maximally singular”. They admit a particularly nice representation as determinants of (relatively) small size — and for a given hypersurface it is a relatively simple matter to check this property. K. Saito originally introduced the concept and proved that discriminants in versal deformations of isolated complete intersection singularities are free divisors. Several other classes of free divisors have been exhibited in the theory of hyperplane arrangements (Terao, Yuzvinsky), deformation theory (Damon, Mond, van Straten), as well as in the theory of Frobenius manifolds (Givental, Hertling).

Expanding on an argument by van Straten, I will explain the ubiquity of free divisors in deformation theory, how they allow to reconstruct the critical locus, and then will add new examples, such as discriminants in

Hilbert schemes of smooth surfaces or discriminants of Gorenstein surface singularities in codimension three (joint with Ebeling).

Time permitting we will comment on other ways of representing discriminants through maximal Cohen-Macaulay modules.

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PIERRETTE CASSOU-NOGUES

BIRATIONAL MORPHISMS FROM  $C^2$  TO  $C^2$

In this talk we will try to explain some work in progress with Peter Russell.

It is known that any configuration of polynomial curves is the set of non properness of morphisms from  $C^2$  to  $C^2$ . One can ask which configuration is the set of missing curves of birational morphisms. Very few is known on the subject. We will try to describe some methods to investigate this problem.

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JAMES DAMON

UNIVERSITY OF NORTH CAROLINA

THE LOCAL AND GLOBAL GEOMETRY OF REGIONS IN  $R^n$  VIA MEDIAL STRUCTURES

A Medial structure  $(M, U)$  consists of a special type of Whitney stratified set  $M$  in  $R^n$ , together with an  $R^n$ -multivalued vector field  $U$  on  $M$ . For example, such a structure arises from the Blum medial axis associated to a region  $W$  with generic smooth boundary  $B$ . We introduce geometric “radial and Edge shape” operators for medial structures which capture the “radial geometry” of  $U$ . We explain how these geometric operators determine the local and relative geometry of  $B$  via a fibration of  $W$  defined by a radial flow from  $M$  along  $U$ .

Furthermore, the global geometry of both  $W$  and  $B$  can be expressed as integrals over  $M$  involving these geometric operators. This leads to results such as a generalization of Weyl’s volume of tubes formula but for generic regions, a medial version of the Gauss-Bonnet theorem, and a relation of the local density of  $M$  at points with limiting values for local average flux integrals.

We will describe the basic ideas and indicate several of the applications.

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WOLFGANG EBELING  
UNIVERSITÄT HANNOVER

INDICES OF 1-FORMS ON SINGULAR VARIETIES

Several notions of an index of an isolated singular point of a 1-form on a germ of a complex analytic variety are discussed: the index (analogue of the GSV-index for vector fields, defined by the speaker and S. M. Gusein-Zade) for a holomorphic 1-form on an isolated complete intersection singularity, the radial index (for a 1-form on the germ of a singular variety), and the homological index of a holomorphic 1-form on the germ of a complex analytic variety with an isolated singularity (inspired by X. Gomez-Mont and G.-M. Greuel). For holomorphic 1-forms on isolated complete intersection singularities, the index coincides with the homological index. Subtracting from the homological index the radial one, one gets an invariant of the singularity which does not depend on the 1-form. For isolated complete intersection singularities this invariant coincides with the Milnor number. This invariant is computed for arbitrary isolated curve singularities and compared with the Milnor number introduced by R.-O. Buchweitz and G.-M. Greuel for such singularities. For the differential of a holomorphic function, the radial index is related to the Euler characteristic of the Milnor fibre of the function. A connection between the radial index and the local Euler obstruction of a 1-form is described. This gives an expression for the local Euler obstruction of the differential of a function in terms of Euler characteristics of some Milnor fibres. This is a report on joint work with S. M. Gusein-Zade and J. Seade.

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STANISLAW JANECZKO  
TECHNICAL UNIVERSITY OF WARSAW

SEARCH FOR INVARIANTS OF SYMPLECTIC SINGULARITIES

Singularities of objects in symplectic space appear through the various disciplines of physics, natural sciences and mathematics itself. There are nonobvious geometrically invisible discrete symplectic invariants of singularities of curves and surfaces (Arnol'd symplectic "ghost") which naturally remain from the symplectic structure at the singular point. We present two

approaches to construct local algebras representing these invariants. One (on the basis of common work with G. Ishikawa) by the symplectic defect and providing the complete classification of simple bifurcations of curves and determine their possible differential and symplectic invariants. And another one (on the basis of common work with W. Domitrz and M. Zhitomirskii) by the method of algebraic restriction to the singular germ and studying the relative characteristic classes of the symplectic structure with zero algebraic restriction. The general problem of symplectic bifurcations of varieties with some relation to elementary models in physics and biology will be considered and studied by isotropic liftings.

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ANATOLY LIBGOBER  
UNIVERSITY OF ILLINOIS

HOMOTOPY GROUPS OF THE COMPLEMENTS TO AMPLE DIVISORS ON  
PROJECTIVE MANIFOLDS

Certain homotopy groups of the complements to divisors with isolated non normal crossings on a projective variety have an algebro-geometric significance. I will discuss vanishing results for these homotopy groups, the module structure over the fundamental group in the case of non vanishing and the connection between the homotopy groups and the geometry of the the set of non normal crossings. I also will discuss applications to the arrangements of hyperplanes.

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DAVID B. MASSEY  
NORTHEASTERN UNIVERSITY

INTERSECTION COHOMOLOGY, MONODROMY, AND THE MILNOR FIBER

Milnor's work on complex hypersurfaces appeared 36 years ago. Goresky and MacPherson introduced intersection homology and cohomology 20 to 25 years ago. Our own work on non-isolated hypersurface singularities began 20 years ago. Despite these facts, we have failed, until now, to notice a simple relation between intersection cohomology, Milnor monodromy, and the Milnor fiber of a hypersurface with a special type of one-dimensional critical locus. We will describe this relation here.

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ALEJANDRO MELLE HERNANDEZ  
UNIVERSIDAD COMPLUTENSE MADRID

A POWER STRUCTURE OVER THE GROTHENDIECK RING OF VARIETIES

Let  $\mathcal{R}$  be either the Grothendieck semiring (semigroup with multiplication) of complex quasi-projective varieties, or the Grothendieck ring of these varieties, or the Grothendieck ring localized by the class  $\Lambda$  of the complex affine line. We define a power structure over these (semi)rings. This means that, for a power series  $A(t) = 1 + \sum_{i=1}^{\infty} [A_i] t^i$  with the coefficients  $[A_i]$  from  $\mathcal{R}$  and for  $[M] \in \mathcal{R}$ , there is defined a series  $(A(t))^{[M]}$ , also with coefficients from  $\mathcal{R}$ , so that all the usual properties of the exponential function hold. In the particular case  $A(t) = (1-t)^{-1}$ , then  $(A(t))^{[M]} = \zeta_{[M]}(t)$  where

$$\zeta_{[M]}(t) := \sum_{k=0}^{\infty} [S^k M] \cdot t^k = 1 + [S^1 M] \cdot t + [S^2 M] \cdot t^2 + [S^3 M] \cdot t^3 + \dots,$$

is the motivic zeta function introduced by M. Kapranov, being  $S^k M$  the  $k$ -th symmetric power  $M^k/S_k$  of the variety  $M$  and  $S_k$  the symmetric group of permutations on  $k$  elements.

Consider the groups  $1 + (t)\mathcal{R}[[t]]$  (under multiplication) and  $(t)\mathcal{R}[[t]]$  (under addition). Using the properties of the power structure over  $1 + (t)\mathcal{R}[[t]]$  we define group isomorphisms  $Lo : 1 + (t)\mathcal{R}[[t]] \rightarrow (t)\mathcal{R}[[t]]$  and  $Ex : (t)\mathcal{R}[[t]] \rightarrow 1 + (t)\mathcal{R}[[t]]$ , one inverse each other, which verify some typical properties of the log and exp functions.

Let  $M$  be a complex projective algebraic surface. The Hilbert scheme  $\text{Hilb}^n M$  of points on  $M$  is a parameter variety for finite subschemes of length  $n$  on  $M$ . It is a resolution of singularities of the singular variety the  $n$ -fold symmetric power  $S^n M$  of  $M$ . As an application we express the generating function of  $\text{Hilb}^n M$  as a exponential of  $M$ . This result fits with the previous results of J. Cheah, L. Goettsche, and W. Soergel.

This is a joint work with S.M. Gusein-Zade and I.Luengo. (Math. Research Letters **11** 2004)

DAVID MOND  
UNIVERSITY OF WARWICK

MILNOR AND TJURINA NUMBERS FOR MATRIX SINGULARITIES

In order to understand the deformations of determinants and Pfaffians resulting from deformations of matrices, we study the deformation theory of composites  $f \circ F$ , with isolated singularities, where  $f$  is a function with (possibly non-isolated) isolated singularity and  $F$  a map into the domain of  $f$ , and we deform  $F$  only. We identify the corresponding  $T^1(F)$  as (something like) the cohomology of a derived functor, and construct a canonical long exact sequence from which it follows that

$$\tau = \mu(f \circ F) - \beta_0 + \beta_1,$$

where  $\tau$  is the length of  $T^1(F)$  and  $\beta_i$  is the length of  $Tor_i(\mathcal{O}_Y/J_f, \mathcal{O}_X)$ . This explains numerical coincidences observed in lists of simple matrix singularities due to Bruce, Tari, Goryunov, Zakalyukin and Haslinger. When  $f$  has Cohen-Macaulay singular locus (as when  $f$  is the determinant function, for example), we obtain relations between  $\tau$  and the rank of the vanishing homology of the zero locus of the composite.

This is a joint work with Victor Goryunov.

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ANDRAS NEMETHI  
OHIO STATE UNIVERSITY

THE SEIBERG-WITTEN INVARIANT CONJECTURE AND PROJECTIVE  
PLANE CURVES

In 2002 L. Nicolaescu and the speaker formulated a very general conjecture which relates the geometric genus of a Gorenstein surface singularity with rational homology sphere link with the Seiberg-Witten invariant (or one of its candidates) of the link. Recently, I Luengo, A. Melle-Hernandez and the speaker found some counterexamples using superisolated singularities. The theory of these hypersurface singularities is equivalent with the theory of cuspidal projective plane curves. In the case when the corresponding curve has only one singular point we were not able to find any counterexample. In

fact, in this case the above Seiberg-Witten conjecture led us to a very interesting and deep property of these curves (generalizing the Seiberg-Witten invariant conjecture, and sitting deeply in algebraic geometry) which seems to generalize famous conjectures and properties (e.g. the Noether-Nagata or the log Bogomolov-Miyaoka-Yau inequalities).

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VITOR HUGO JORGE PEREZ  
ICMC-USP

LÊ-IOMDINE-VOGEL FORMULA, SEGRE NUMBERS, INTEGRAL  
DEPENDENCE AND HYPERSURFACE SINGULARITIES

Massey defines the Lê-Vogel cycles and numbers, and obtains the Lê-Iomdine-Vogel formulas. We prove that these formulas can be written in terms of Segre numbers of an ideal with no-finite colength and multiplicities of an ideal of finite colength. With these formulas we prove generalizations of various results involving the multiplicities and Lê numbers given by Massey and Gaffney. We also obtain a relationship between mixed Segre numbers of ideals and mixed multiplicities of ideals of finite colength. These results are applied to the study of integral dependence of ideals and equisingularity. Finally we give a quick application of our result to Whitney equisingularity of map germs from  $(\mathbb{C}^3, 0) \rightarrow (\mathbb{C}^3, 0)$ . Greuel shows that the constancy of the Milnor number of families of hypersurfaces with isolated singularity is guaranteed by a condition of the integral closure of the Jacobian ideal, we give a similar result for families with one dimensional singular set.

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MARIA APARECIDA SOARES RUAS  
ICMC-USP

$M$ - DEFORMATIONS OF  $\mathcal{A}$ -SIMPLE GERMS

We investigate properties of  $\mathcal{A}$ -simple map-germs from  $\mathbb{R}^n, 0$  into  $\mathbb{R}^p, 0$ ,  $n \geq p$ .

The main result is that all  $\mathcal{A}$ -simple singularities of minimal corank (i.e. of corank  $n - p + 1$ ) have an  $M$ -deformation, that is a deformation in which the maximal numbers of isolated stable singular points are simultaneously present in the discriminant.

This is a report on joint work with J. Rieger.

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WIM VEYS  
KATHOLIEKE UNIVERSITEIT LEUVEN

ON THE POSSIBLE POLES OF TOPOLOGICAL AND RELATED ZETA  
FUNCTIONS

The complex singular index or log canonical threshold is a classical singularity invariant, associated to (the germ at the origin of) a complex polynomial function  $f$  on  $C^n$ . The question which rational numbers can occur as values for this invariant was studied intensively. We consider the finite set of poles of the topological and related zeta functions of  $f$  as a finer invariant, and we attack the same question. We determine for instance all possible smallest poles when  $n = 2$  or  $3$ ; for  $n = 2$  this is a generalization of the fact that the log canonical threshold is never in the interval  $]5/6, 1[$ . We also obtain an optimal lower bound.

This is a joint work with my student Dirk Segers.

# 3. Propositions of Talks

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FUENSANTA AROCA  
ICMC USP

VALUATIONS COMPATIBLE WITH A PROJECTION

Let  $\mathcal{H}$  be an irreducible affine hypersurface embedded in  $\mathbb{C}^{N+1}$ , let  $\pi : \mathcal{H} \rightarrow \mathbb{C}^N$  be a finite projection and denote by  $\mathcal{R}$  be the ring of polynomials in  $N$  variables with complex coefficients. Given valuation  $\nu : \mathcal{R} \rightarrow \mathbb{R}_{\geq 0} \cup \{\infty\}$ ; we want to describe all the valuations  $\tilde{\nu} : \mathcal{O}_{\mathcal{H}} \rightarrow \mathbb{R}_{\geq 0} \cup \{\infty\}$  that extend  $\pi_*\nu$ . That is, that make the diagram

$$\begin{array}{ccc} \mathcal{O}_{\mathcal{H}} & \xrightarrow{\tilde{\nu}} & \mathbb{R}_{\geq 0} \cup \{\infty\} \\ & \swarrow \pi^* & \searrow \nu \\ & \mathcal{R} & \end{array} \quad (3)$$

commute.

We will describe these valuations when  $\nu$  is a monomial valuation whose weight vector is not orthogonal to any of the faces of the Newton Polyhedron of the discriminant of the projection.

This description is done in terms of the Puiseux parameterizations with exponents in a cone introduced in [1]. The question was posed to me by Bernard Teissier at the congress "Singularity theory and Applications" held at Sapporo in 2003.

## References

- [1] John McDonald. Fiber polytopes and fractional power series. *J. Pure Appl. Algebra*, 104(2):213–233, 1995.

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DANIEL BARLET  
UNIVERSITÉ DE NANCY

MODULE DE BRIESKORN A L'ORIGINE POUR UNE CLASSE  
D' HYPERSURFACES A LIEU SINGULIER DE DIMENSION 1

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VALENTINA BARUCCI  
UNIVERSITÀ LA SAPIENZA

THE APERY ALGORITHM FOR A PLANE CURVE SINGULARITY WITH TWO  
BRANCHES

Let  $R = \mathbf{C}[[X, Y]]/(F)$  be an irreducible plane algebroid curve (a branch) and let  $v(R) = S$  be its value semigroup. If  $R'$  is the ring obtained by blowing up the maximal ideal of  $R$ , then, by a result of Apéry, the semigroups  $v(R)$  and  $v(R')$  are strictly related: there is a formula to get a particular generating set, called the Apéry set, for  $v(R')$  from that of  $v(R)$  and vice versa. This does not happen in general for non plane branches and is the reason why for plane branches the semigroup characterizes as well as the multiplicity sequence a class of equivalence. By Apéry's result, one can get the 0 semigroup from the multiplicity sequence and vice versa.

In the joint paper with M. D'Anna and R. Fröberg "The Apéry algorithm for a plane curve singularity with two branches" (accepted for publication on *Breiträge zur Algebra und Geometrie*), we generalize these results to the case of a plane curve with two branches. As an application of the main theorem, we get the multiplicity tree from the semigroup and vice versa. By a numerical characterization of a multiplicity tree of a plane curve with two branches, we get also a constructive characterization of the two branches plane curve semigroups.

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MARIA ALICE BERTOLIM  
UNIVERSIDADE ESTADUAL DE CAMPINAS

MINIMAL MORSE FLOWS ON COMPACT MANIFOLDS

In this talk we compute the minimal number of non-degenerate singularities that can be realized on some manifold with non-empty boundary in terms only of abstract homological boundary information. We specify the index and the types (connecting or disconnecting) of the singularities realizing the minimum. The Euler characteristics of manifolds realizing the minimum are obtained and the associated Lyapunov graphs of Morse type are described and shown to have the lowest topological complexity.

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CARLES BIVIÀ-AUSINA  
UNIVERSITAT POLITÈCNICA DE VALÈNCIA

THE INTEGRAL CLOSURE OF MODULES AND NEWTON POLYHEDRA

The computation of the integral closure of an ideal or a submodule is a central problem in commutative algebra. Moreover, the solution of this problem has applications to singularity theory, by virtue of the works of Teissier and Gaffney on the equisingularity of deformations of hypersurface and isolated complete intersection singularities, respectively. We compute the integral closure and the Buchsbaum-Rim multiplicity of a wide class of submodules of  $\mathcal{O}_n^p$  through suitable Newton polyhedra. This class constitutes an extension of Newton non-degenerate ideals in the sense of Saia. Considering a theorem of Rees on reductions of submodules, we also characterize the submodules of the above-mentioned class through a numerical expression for its Buchsbaum-Rim multiplicity.

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SOFIA CASTRO  
FACULDADE DE ECONOMIA DO PORTO AND CENTRO DE MATEMÁTICA  
UNIVERSIDADE DO PORTO

INTRINSIC COMPLETE TRANSVERSALS AND THE RECOGNITION OF  
BIFURCATIONS

Let  $G$  be a Lie group acting smoothly on an affine space  $A$  and let  $W$  be a vector subspace of  $V_A$ . A vector subspace  $T$  of  $W$  is a *complete transversal* if it is transversal to the orbit of  $x_0 \in A$  and meets each orbit through the affine space  $x_0 + W$  of  $A$ .

Complete transversals were used by Bruce, Kirk and du Plessis (Complete transversals and the classification of singularities, *Nonlinearity* **10**, 253–275 (1997)), together with some properties of unipotent algebraic groups (see Bruce, du Plessis and Wall, Determinacy and unipotency, *Invent. Math.* **88**, 521–554 (1987)), to classify singularities of map-germs with respect to a range of equivalence relations.

We show that the concept of complete transversal can be used to classify bifurcation problems, with or without symmetry. In many cases a complete transversal can be chosen, which is invariant under a large subgroup of the

group of equivalences. This allows the recognition problem to be solved systematically. We illustrate this with some examples.

This is a joint work with Andrew du Plessis, U. Aarhus, Denmark.

## References

[1] J. W. Bruce, N. P. Kirk and A. A. du Plessis, Complete transversals and the classification of singularities, *Nonlinearity* **10**, 253–275 (1997).

[2] J. W. Bruce, A. A. du Plessis and C. T. C. Wall, Determinacy and unipotency, *Invent. Math.* **88**, 521–554 (1987).

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ANDRE DIATTA  
LIVERPOOL

SYMMETRY SETS AND MEDIAL AXES OF PLANE SECTIONS OF SMOOTH SURFACES. THE PATTERNS OF VERTICES AND INFLEXIONS ON FAMILIES OF PLANE CURVES.

The symmetry set (SS) of a plane curve, is the closure of the set of centres of circles which are tangent to the curve at two different places, at least. The medial axis (MA) is the subset of the SS consisting of the closure of the locus of centres of circles which are maximal, i.e whose radii are the minimum distance from their centres to the curve. These two geometric objects are designed to capture the features and properties of a curve, or a more generally a shape. They show to be very useful in Medical imaging, Computer Vision, ...

We are interested in local transitions of the SS/MA in 1-parameter families of plane curves (such as isophote curves) obtained as plane sections of a smooth surface in the 3-space.

Symmetry sets of 1-parameter families of smooth plane curves were classified by J.W. Bruce and P. J. Giblin. But their results (and the methods used from Singularity theory) do not apply any more, when the family includes singular curves, as is the case when the plane sections is tangent to the surface so that this section is a singular curve.

In this work, we adopt a very direct approach which consists of first keeping track on the evolution of some crucial facts on the SS/MA. More precisely we will trace the patterns of vertices (maxima and minima of curvature) and

inflexions on the sections of a surface as the section passes through a tangential point. The vertices are crucial to the understanding of the symmetry set since it has branches which end at the centres of curvature at vertices. Thus from the way in which vertices behave we can deduce a good deal about the evolution of the symmetry set and its local number of branches. The inflexions correspond to where the evolute of the curve, goes off to infinity. We may assume that our surface is locally given by an equation  $z = f(x, y)$  for some smooth function  $f$ . This comes to an interesting problem of Geometry and Singularity Theory, about vertices and inflexions on a curve  $z = c$  as well as the geometry of their patterns as  $c$  varies. By studying the geometry of the patterns, we are able to classify all possible scenarii of how vertices and inflexions are distributed along the curves  $f = c$ , depending on different cases. We also keep track on the evolution of the curvature of the curves  $f = c$  at vertices and control its limit when the vertices collap at singular points.

Some other special points on SS, the so-called  $A_1A_2$ ,  $A_1^3$ , ... points, are also carefully studied.

Last, using parametrised surfaces, we produce example of Symmetry Sets and Medial Axes illustrating each case.

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PETER DONELAN

VICTORIA UNIVERSITY OF WELLINGTON, NEW ZEALAND

TRAJECTORY SINGULARITIES FOR A CLASS OF PARALLEL MOTIONS

A rigid body, three of whose points are constrained to move on the coordinate planes, has three degrees of freedom. Bottema and Roth showed that there is a point whose trajectory is a solid tetrahedron, the vertices representing codimension 3 singularities. A theorem of Gibson and Hobbs says that, for general 3-parameter motions, such singularities do not occur generically. However motions subject to this kind of constraint arise as interesting examples of parallel motions in robotics and we show that, within this class, such singularities occur stably. (joint work with Chris Gibson and Matthew Cocks)

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DANIEL DREIBELBIS

THE GEOMETRY OF FLECNODAL NORMALS

Given an immersed manifold  $M$  in Euclidean space, a normal vector  $\mathbf{v}$  at a point  $p$  is a *flecnodal normal* if the manifold, when projected into the subspace  $T_pM \oplus \mathbf{v}$ , has four-point contact with a line at  $p$ . This definition is a generalization of the flecnodal curve on surfaces in 3-space. In this talk, we will justify why flecnodal normals are worthy of study. We will look at the structure of flecnodal normals, both as a submanifold of the normal bundle and its image in the Gauss map. Also, we will link the structure of the flecnodal normals to other geometric features of the manifold, such as inflection points and bitangencies. Surfaces immersed in 4-space will receive special attention.

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MASSIMO FERRAROTTI

APPROXIMATION OF SUBANALYTIC SETS BY NORMAL CONES.

joint work with E.Fortuna and L.C.Wilson

In a previous paper we introduced local equivalence of two sets at a point: two sets are equivalent of order  $s$  at  $p$  if the Hausdorff distance of their sections with spheres of center  $p$  and radius  $r$  is  $o(r^s)$  for  $s > 0$ . In the present work we generalize this notion to the notion of equivalence of two sets along a common stratum  $X$ , taking tubes of center  $X$  and radius  $r$  instead of spheres, and we prove that, under suitable regularity hypothesis (e.g.  $w$ -regularity), a subanalytic set is 1-equivalent to its normal cone at  $X$ .

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JACQUES FURTER

SYMMETRY BREAKING BIFURCATION AND SINGULARITY THEORY

Many bifurcation problems share their core (or organising centre), the singularity ignoring the parameters. The differences between the problems lie in the parameters, symmetry or even unfolding structures. Path formulation provides a concept that helps to consider efficiently the different possibilities. We discuss examples from applications to elasticity and mechanics that have a rich symmetry breaking structure. In some examples there is a dialectic between the modelling and singularity theory, one informing on the other.

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TERENCE GAFFNEY  
NORTHEASTERN UNIVERISTY

MULTIPLICITY OF PAIRS OF MODULES AND HYPERSURFACE  
SINGULARITIES

In [1] and [2], Gaffney introduced the multiplicity of a pair of modules as a new tool in equisingularity theory. The invariants introduced using this tool have the advantage that they must be independent of the parameters in the family when the stratification condition they describe holds. These invariants provide a framework for studying the equisingularity conditions  $W$ ,  $W_f$  and  $A_f$  for very general families of spaces and functions. In this talk we will illustrate the use of these invariants in the study of families of functions with non-isolated singularities and show how the invariants arise naturally in the work of Pellikaan and other students of Siersma.

[1] T. Gaffney, *Generalized Buchsbaum-Rim Multiplicities and a Theorem of Rees* Communications in Algebra, 31 (2003) 3811-3828.

[2] T. Gaffney, *Polar methods, invariants of pairs of modules and equisingularity*, Real and Complex Singularities (Sao Carlos, 2002), Ed. T.Gaffney and M.Ruas, Contemp. Math.,#354, Amer. Math. Soc., Providence, RI, June 2004, 113-136.

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PETER GIBLIN  
UNIVERSITY OF LIVERPOOL

LOCAL FEATURES IN VIEWS OF ILLUMINATED SURFACES

We apply methods of singularity theory to a problem from computer imaging concerning the stable configurations and transitions under variation of viewpoint of the interaction between shade-shadow curves, apparent contours, and geometric surface features such as crease-edges corners, boundary-edges, and surface markings. We concentrate on the case of "stable lighting", for which properties of shade-shadow curves are stable under slight perturbations of the light source direction.

We show that these interactions can be classified by a group of equivalences which are geometric subgroups of the right-left group  $A$  in suitable

cases. Hence, we may use all of the traditional results of singularity theory to determine the stable configurations, classify the unstable configurations, and determine their versal unfoldings. In doing this we are able to make use of a number of previous classifications for apparently different situations. We further show that there are geometric obstructions for realizing certain abstract germs as illuminated surfaces and for realizing versal unfoldings through movement of viewer direction.

This is a joint work with James Damon (UNC Chapel Hill) and Gareth Haslinger (Liverpool), funded by European Union research grant INSIGHT2+.

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VINCENT GRANDJEAN  
UNIVERSIT DE BATH

ON THE ASYMPTOTIC GEOMETRY OF REAL POLYNOMIALS THROUGH  
GRADIENT TRAJECTORIES

Given a real polynomial function  $f$  defined over  $\mathbb{R}^n$ , from Thom we know the levels  $f^{-1}(c)$  describe finitely many topological types. A value at which the topology is changing is called the bifurcation values. Any critical value is a bifurcation value, but a bifurcation value can also be a regular value. Thom did not provide any description of these bifurcation values that are also regular values. Nevertheless in the early eighties, the work of several authors led to define the notion of asymptotic critical value of a polynomial (real or complex). These values form a finite set  $K_\infty(f)$  and are the values at in a neighbourhood of which the Malgrange condition is not satisfied. It was proved that any bifurcation value that was a regular value, was also an asymptotic critical value, so that we then got an idea where to find the regular bifurcation values. In this work, using a new gradient-like inequality in a neighbourhood at infinity of  $f^{-1}(c)$  and its related exponent  $\rho_c$ , I will show that the bifurcation values that are regular have to be looked for only at the regular asymptotic critical values  $c$  for which  $\rho_c = 1$  (the Malgrange condition is satisfied if and only if  $\rho_c \leq 0$ ). Next, I will come more closely to the real plane case, where having  $\rho_c = 1$  is in fact equivalent to several other conditions :

- having the levels neighbouring  $c$  provided with a richer asymptotic geometry than the level  $c$  itself, which can be expressed in terms of polar curves,

- the condition  $S_f$ , that is tangency to the euclidean spheres infinitely near  $f^{-1}(c)$ ,
- the discontinuity of the limit of absolute curvature when getting closer and closer to the levels  $c$ .

This is a partially joint work with Didier D'ACUNTO (Universidad Complutense, Madrid).

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MARCELO ESCUDEIRO HERNANDES  
UNIVERSIDADE ESTADUAL DE MARINGÁ  
SPECIALS GAPS DIAGRAM OF PLANE BRANCHES

In this talk we introduced the special gaps diagram, that is an invariant finer than Tjurina number with respect to the analytic equivalence of plane branches and we characterized all possible admissible diagrams that are associated to the plane branches with a fixed semigroup  $\langle n, m \rangle$ .

This is a joint work with Abramo Hefez.

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CLAUS HERTLING  
IECN, UHP NANCY 1, FRANCE  
BERNOULLI MOMENTS OF SPECTRAL NUMBERS

The distribution of the spectral numbers of an isolated hypersurface singularity is studied in terms of the Bernoulli moments. These are certain rational linear combinations of the higher moments of the spectral numbers. They are related to the generalized Bernoulli polynomials. We conjecture that their signs are alternating and prove this in many cases. This generalizes a conjecture on the variance of the spectral numbers, which was proposed at the Sao Carlos workshop 2000. One motivation for the Bernoulli moments comes from the comparison with compact complex manifolds and their Chern classes.

This is a joint work with Thomas Brélivet, see math.AG/0405501.

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KEVIN HOUSTON  
UNIVERSITY OF LEEDS

WHITNEY EQUISINGULARITY AND DISENTANGLEMENTS

For a family of differentiable maps one would like to determine whether the family is in some sense trivial. A useful notion is Whitney equisingularity as this implies that the maps are topologically equivalent. The relationship between some easily defined topological invariants of the family members and Whitney equisingularity is investigated for corank 1 complex analytic maps between  $n$ -space and  $(n+1)$ -space, and in particular the surface to 3-space case.

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LEÓN KUSHNER  
UNAM

TOPOLOGY AND GEOMETRY OF QUASIHOMOGENEOUS POLYNOMIALS

We consider the space of quasihomogeneous polynomials (*qh.p.*), in two variables of weights  $(\frac{1}{3}, \frac{1}{6})$ . The group acting in this 4 dimensional vector space can be seen as the invertible triangular matrices  $\begin{pmatrix} a & b \\ 0 & d \end{pmatrix}$ , and the product is given by

$$\begin{pmatrix} a & b \\ 0 & d \end{pmatrix} \cdot \begin{pmatrix} e & f \\ 0 & g \end{pmatrix} = \begin{pmatrix} ae & af + bg^2 \\ 0 & dg \end{pmatrix}$$

The complexified stabilizers of the orbits together with a stability condition separates the relative finite determined *qh.p.* with the ones they are not. We observe that the orbits are at most 3-dimensional in a 4-dimensional space.

In the space of homogeneous polynomials, of two variables and degree three, the orbits of highest dimension are open and finite determined. In our space, the highest dimension orbits are not open because, as we said, the dimension is three. Another main difference is that not all of these orbits are finite determined.

We also compare the right-determination with the right relative determination, that is to say, the germs of diffeomorphisms with the canonical action with the germs of diffeomorphisms of the form:

$$(x, y) \mapsto (ax + by^2 + h_1(x, y), dy + h_2(x, y))$$

where  $h_1 \in m^3(2)$  and  $h_2 \in m^2(2)$ .

This is a joint work with Radmila Bulajich and Santiago López de Medrano.

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ISABEL LABOURIAU  
INVARIANTS FOR BIFURCATION

Bifurcation problems with one parameter are studied here. We develop a method for computing a topological invariant, the number of fold points in a stable one-parameter unfolding for any given bifurcation of finite codimension. We introduce another topological invariant, the algebraic number of folds. The invariant gives the number of complex solutions to the equations of fold points in a stabilization, an upper bound for the number of fold points in any unfolding. It can be computed by algebraic methods, we show that it is finite for germs of finite codimension. An open question is whether this value is always attained as the maximum number of fold points in a stable unfolding. This is joint work with M.A.S.Ruas

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DANIEL LEHMANN  
UNIVERSITÉ DE MONTPELLIER

DICRITICAL SINGULARITIES OF HOLOMORPHIC VECTOR FIELDS

Dicritical singularities of holomorphic vector fields give rise to specific indices whose sum has a topological interpretation.

This is a joint work with C. Camacho.

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DANIEL LEVCOVITZ  
ICMC-USP

DIFFERENTIAL SIMPLICITY IN POLYNOMIAL RINGS AND ALGEBRAIC  
INDEPENDENCE OF POWER SERIES

Let  $k$  be a field of characteristic zero,  $f(X, Y), g(X, Y) \in k[X, Y]$ ,  $g(X, Y) \notin (X, Y)$  and  $d := g(X, Y)\frac{\partial}{\partial X} + f(X, Y)\frac{\partial}{\partial Y}$ .

We establish a connection between the  $d$ -simplicity of the local ring  $k[X, Y]_{(X, Y)}$  and the transcendency of the solution in  $tk[[t]]$  of the algebraic differential equation  $g(t, y(t))\frac{\partial}{\partial t}y(t) = f(t, y(t))$ . We use this connection to obtain some interesting results in the theory of the formal power series and to construct new examples of differentially simple rings.

This is a joint work with Paulo Brumatti (UNICAMP) and Yves Lequain (IMPA).

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BERND MARTIN

MODULAR SPACES OF DEFORMATION FUNCTORS

The notion of a modular space has been introduced for complete complex varieties and for analytic polyhedron by Palamodov, and in a formal context by Laudal, cf. [P1], [P3], [L]. It is a possible approach of constructing a kind of moduli of isolated singularities by restricting the versal family to subgerms which have an universal property at least for all families induced from it.

The author has obtained some progress in calculating non-trivial examples of modular deformations at least for complete intersection singularities and for space curve singularities by connecting the modular property with flatness of the relative Tjurina module and by finding an obstruction calculus of lifting flatness, cf. [M1], [M2]. The computations are done in the SINGULAR computer algebra system, cf. [S].

Here we want to present further developments of this concept obtained by T.Hirsch and the author, started in [HM]. Various characterizations of the modular property are extended for different deformation functors. New examples of modular strata with special properties are presented.

The results on modular strata are unified for deformations of isolated singularities containing a fixed fat point. This concept includes ordinary deformations, deformations with section, deformations with constant embedding dimension and equimultiple deformations. A modular subgerm  $M \subseteq S$  of the base space of a miniversal deformation  $X \rightarrow S$  is characterized by the following equivalent conditions:

- injectivity of the relative Kodaira-Spencer map  $T^0(S, CO_m) \rightarrow T^1(X/S)_{|M}$ ,
- lifting property of vector fields of the special fiber:  $T^0(X_{|M}/M) \rightarrow T^0(X_0)$  is surjective,
- flatness of the extended relative Tjurina module  $\tilde{T}^1(X/S) \otimes_{CO_S} CO_M$ .

Moreover, if the deformations of  $X_0$  are unobstructed, then modularity is indeed equivalent to the flatness of the relative Tjurina module. Examples, computed so far include:

- modular families of hypersurface singularities with a splitting singular locus (not possible for a  $\mu$ -constant family),
- modular strata of all unimodular singularities,
- most singularities with  $\mu - \tau \leq 2$ ,
- some very special modular families of Artinian complete intersection singularities.

## References

- [L] O.A. Laudal, *Formal moduli of algebraic structures*, Lecture Notes in Math. **754**, Springer-Verlag, Berlin-New York, (1979).
- [HM] T. Hirsch, B. Martin, *Deformations with sections: Cotangent cohomology, flatness and modular subspaces*, preprint [math.CV/0306281](#).
- [M1] B. Martin, *Algorithmic computation of flattenings and of modular deformations*, J. Symbolic Comput. **34** (2002), no. 3, 199-212.
- [M2] B. Martin, *Modular deformation and Space Curve Singularities*, Rev. Mat. Iberoamericana **19** (2003), no. 2, 613-621.

- [P1] V.P. Palamodov, *Moduli and versal deformations of complex spaces*, in: Varieties analytiques compactes, LNM **683** Springer-Verlag, Berlin-New York, (1978), 74-115.
- [P3] V.P. Palamodov, *Deformation of analytic polyhedrons*, J. Algebraic Geometry **2**, (1993), 263-294.
- [S] G.-M. Greuel, G. Pfister, H. Schnemann, SINGULAR 2.0, University of Kaiserslautern, 2002, <http://www.singular.uni-kl.de>.

PETER MORMUL  
WARSAW UNIVERSITY

SINGULARITY CLASSES OF SPECIAL  $k$ -FLAGS

Special multi-flags directly generalize (1-)flags that are generated by Goursat distributions: the dimensions of consecutive Lie squares grow now by  $k \geq 2$  instead of 1. And ‘special’ is a necessary technical addition, void for Goursat. (Without it multi-flags can be extremely complicated, as already are Cartan’s, [3], classical (3, 5) distributions being explained only now in the vast Agrachev–Zelenko theory [1].)

Recalling, for any fixed length  $r \geq 2$ , Goursat flags have been stratified into  $2^{r-2}$  invariant classes (called Kumpera–Ruiz by the authors of [5]). Now, [6], special 2-flags are being stratified ( $r \geq 3$ ) into

$$2 + 3 + 3^2 + \dots + 3^{r-2} \quad (4)$$

invariant singularity classes; special 3-flags ( $r \geq 4$ ) into

$$2^{r-1} + (2^{r-2} - 1)4^0 + (2^{r-3} - 1)4^1 + \dots + (2^1 - 1)4^{r-3} \quad (5)$$

singularity classes, and so on; the precise recurrence amongst these numbers is a bit involved (but clear). To give a fuller idea of the sizes, here are, for the length  $r = 7$ , the numbers of different singularity classes of special  $k$ -flags, for  $k \in \{1, 2, \dots, 6\}$  [the value  $32 = 2^{7-2}$  is still from the Goursat world]:

$k$	1	2	3	4	5	6
#	32	365	715	855	876	877

(365 and 715 are the values of (1) and (2), resp., for  $r = 7$ ).

As regards the local classification of special  $k$ -flags, singularity classes approximate the orbits from above and, albeit numerous, are only rough invariants. Already in length 3 one of them splits up into three orbits, yet the classification in this length is still stable with respect to flag's width  $k \geq 2$ .

Starting from length 4, the local classification ceases to be stable with respect to  $k$ . For instance, there is one singularity class in that length, consisting of six orbits in width 2, and consisting of seven or more orbits in width 3.

We interpret this loss of stability in terms of singularities of curves in  $\mathbf{R}^{k+1}$ , that are closely related to special  $k$ -flags. For ex., singularities of curves in  $\mathbf{R}^3$  ( $k = 2$ , [4]) as contrasted with those in  $\mathbf{R}^4$  ( $k = 3$ , [2]).

### References

- [1] A. A. Agrachev, I. Zelenko; *Geometry of Jacobi curves*, I and II. J. Dynam. Control Syst. **8** (2002), 93-140 and 167-215.
- [2] V.I. Arnold; *Simple singularities of curves*. Proc. Math. Inst. Steklov **222** (2000).
- [3] E. Cartan; *Les systèmes de Pfaff à cinq variables et les équations aux dérivées partielles du second ordre*. Ann. Ecole Normale **27** (1910), 109-192.
- [4] C. G. Gibson, C. A. Hobbs; *Simple singularities of space curves*. Math. Proc. Camb. Phil. Soc. **113** (1993), 297-310.
- [5] R. Montgomery, M. Zhitomirskii; *Geometric approach to Goursat flags*. Annales de l'Inst. H. Poincaré – AN **18** (2001), 459-493.
- [6] P. Mormul; *Geometric singularity classes for special  $k$ -flags,  $k \geq 2$ , of arbitrary length*. Preprint in: Singularity Theory Seminar Vol. 8, S. Janeczko (Ed), Warsaw University of Technology 2003, 87-100.

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HOSSEIN MOVASATI  
UNIVERSITÄT GOETTINGEN

### MIXED HODGE STRUCTURE OF GLOBAL BRIESKORN MODULES

In this talk we introduce the mixed Hodge structure of the Brieskorn module of a polynomial  $f$  in  $n + 1$  variables, where  $f$  satisfies a certain regularity condition at infinity and hence has isoalted singularities. We give an algorithm which produces a basis of the pieces of the mixed Hodge structure

and the Brieskorn module itself. As an application we introduce the notion of a Hodge cycle in regular fibers of  $f$  by vanishing of integrals of certain polynomial  $n$ -forms over topological  $n$ -cycles on the fibers of  $f$ . Since the  $n$ -th homology of a regular fiber is generated by vanishing cycles, this leads us to study Abelian integrals over vanishing cycles. Our result generalizes and uses the arguments of J. Steenbrink 1977 for quasi-homogeneous polynomials.

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ANA CLAUDIA NABARRO

VECTOR FIELDS IN  $\mathbb{R}^2$  WITH MAXIMAL INDEX

This is a joint work with Prof. M.A.S. Ruas

We use Poincaré's method to investigate the index of vector fields in the plane. If  $m$  is the degree of the principal part of the vector field  $X$  at zero, we find necessary and sufficient conditions for the absolute value of the index of  $X$  to be  $m$ . We also describe the geometry of these vector fields.

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REGILENE OLIVEIRA

ON PAIRS OF POLYNOMIAL FOLIATIONS

Pairs of differential 1-forms appear naturally in several mathematical contexts. For example in quadratic differential forms (also known as binary differential equations), Differential Geometry and Partial differential equations. In this work we deal with pairs of planar foliations represented by polynomial differential 1-forms. The main result concern global and local stability as well as the finite determinacy for these pairs.

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ANNE PICHON  
IML, MARSEILLE

SOME GENERALIZATIONS OF MUMFORD'S CRITERION IN THE  
NON-ISOLATED CASE

Let  $(S, p) \subset (\mathbf{C}^n, 0)$  be a germ of complex surface and let  $L(S) = S \cap \mathbf{S}_\epsilon^{2n-1}$  be its link, where  $\mathbf{S}_\epsilon^{2n-1} = \{x \in \mathbf{C}^n / \|x\| = \epsilon\}$  denotes a Milnor ball for  $S$ . When  $S$  is normal, Mumford's result gives a topological characterization of smoothness : if  $L(S)$  has the homotopy type of the 3-sphere, then  $(S, p)$  is the smooth germ.

What happens when the singularity  $(S, p)$  is allowed to be non isolated ?

We explore two different ways in order to generalize Mumford's criterion to  $(S, 0) = (f^{-1}(0), 0)$  where  $f : (\mathbf{C}^3, 0) \rightarrow (C, 0)$  is an analytic germ with a 1-dimensional critical locus.

The first one concerns the boundary of the Milnor fibre of  $f$ . We prove that when  $f$  is irreducible, then  $(S, 0)$  is the smooth germ if and only if the boundary of the Milnor fiber of  $f$  is homeomorphic to  $\mathbf{S}^3$ . This is a joint work with F. Michel.

The second one concerns the link of the normalization of  $(S, 0)$ . We prove Lê's conjecture for a large family of germs  $f : (\mathbf{C}^3, 0) \rightarrow (C, 0)$ , namely if the link of the normalization of  $(S, 0)$  is homeomorphic to  $\mathbf{S}^3$ , then  $f$  is equisingular with one branch. This part is a joint work with I. Luengo and A. Melle.

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ANNA PRATOUSSEVITCH  
BONN

COMBINATORIAL GEOMETRY OF Q-GORENSTEIN QUASI-HOMOGENEOUS  
SURFACE SINGULARITIES

The links of Q-Gorenstein quasi-homogeneous surface singularities can be described as quotients

$$\Gamma \backslash \widetilde{\mathrm{SU}}(1, 1) / \Phi$$

of the simply connected Lie group  $\widetilde{\mathrm{SU}}(1, 1)$  by the action of a discrete subgroup  $\Gamma$  by multiplication on the left and by the action of a cyclic discrete

subgroup  $\Phi$  by multiplication on the right. The Killing form induces a bi-invariant Lorentzian metric of constant curvature on the Lie group  $\widetilde{\text{SU}}(1, 1)$ . We describe the Lorentz space form  $\Gamma \backslash \widetilde{\text{SU}}(1, 1) / \Phi$  by constructing a fundamental domain  $D$  for the action of  $\Gamma \times \Phi$ . We want  $D$  to be a polyhedron with totally geodesic faces. We construct such  $D$  for  $\Gamma$  and  $\Phi$  satisfying certain conditions.

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MARCELO JOSÉ SAIA  
ICMC - USP

THE INTEGRAL CLOSURE OF NEWTON DEGENERATE IDEALS AND  
TOPOLOGICAL TRIVIALITY OF GERMS OF HYPERSURFACES

In the Theorem 1.11 of [1] it is given a necessary and sufficient condition to compute the integral closure of ideals with finite colength in the ring of complex holomorphic germs  $f : (\mathbb{C}^n, 0) \rightarrow \mathbb{C}$  or in the ring of real analytic germs  $f : (\mathbb{R}^n, 0) \rightarrow \mathbb{R}$ . This condition is based in geometric sets associated to the  $(n-1)$ -dimensional compact faces of the Newton polyhedron. We show here that this condition is not necessary, with the following counterexample.

**Example:** Let  $I = \langle g_1, g_2, g_3 \rangle$  in  $\mathcal{O}_3$  with  $g_1 = x^{10} + xy(x-y)^2(5x^2 - y^2)$ ,  $g_2 = y^{11} + x^{11} + xy(x-y)^2(x^2 - 5y^2)$  and  $g_3 = z^7$ . The Theorem 1.11 of [1] asserts that a necessary condition for a monomial  $x^\ell y^m z^n$  to be in the integral closure of the ideal  $I$  is  $\langle (7, 7, 6); (\ell, m, n) \rangle = 7\ell + 7m + 6n \geq 70$ , but the monomial  $z^7$  is in the ideal  $I$ , hence it is in the integral closure of this ideal and it does not satisfy the condition of the item (ii) since  $\langle (0, 0, 7), (7, 7, 6) \rangle = 42$ .

We also show in this note that the method used in the proof of the Theorem 1.11 is valid if we consider an additional hypothesis of non-degeneracy for all  $(r)$ -dimensional compact faces of the Newton polyhedron, with  $r < n - 1$ .

We apply this algorithm to give a sufficient condition for the constancy of the Milnor number of one parameter families of germs of functions  $f_t(x) : (\mathbb{C}^n, 0) \rightarrow (\mathbb{C}, 0)$ , such that the ideals  $\langle x_1 \frac{\partial f_t}{\partial x_1}, \dots, x_n \frac{\partial f_t}{\partial x_n} \rangle$  satisfy this hypothesis for small values of  $t$ .

**References:**

- [1] M. J. Saia, The Integral Closure of Ideals and Whitney equisingularity of germs of hypersurfaces. *Matemática Contemporânea*, S.B.M., IV Workshop in Real and Complex Singularities, ed M.A.S.Ruas, **12**, (1997), 183–198.

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EVGENII SHUSTIN  
TEL AVIV UNIVERSITY

PLANAR DEFORMATIONS OF SPACE CURVE SINGULARITIES

Let  $(\Sigma_0, z)$  be a singular germ of a reduced algebraic surface,  $(C_0, z) \subset (\Sigma_0, z)$  a germ of a reduced algebraic curve, being a Cartier divisor. We study one-parametric deformations  $(\Sigma_t, C_t)$  of the pair  $(\Sigma_0, C_0)$ , such that  $\Sigma_t$ ,  $t \in (\mathbb{C}, 0)$ , is a flat family of surface germs which are non-singular as  $t \neq 0$ , and  $C_t \subset \Sigma_t$ ,  $t \in (\mathbb{C}, 0)$ , is a flat family of curve germs. In particular, we want to compute the *planar  $\delta$ -invariant*  $\delta_{\text{pl}}(C_0, z)$  defined as the maximal number of nodes of a germ  $C_t$ ,  $t \neq 0$ , in the above deformations.

An important example, when  $\Sigma_0 = \Sigma'_0 \cup \Sigma''_0$ , where  $\Sigma'_0$  and  $\Sigma''_0$  are smooth, transversally intersecting surface germs, appears in study of the irreducibility of the varieties of plane curves of given degree and genus, and in the enumerative geometry of nodal curves. In this case  $C_0 = C'_0 \cup C''_0$  with  $(C'_0, z) \subset (\Sigma'_0, z)$  and  $(C''_0, z) \subset (\Sigma''_0, z)$  planar curve germs. In particular, a Ran-Caporaso-Harris lemma states that if  $(C'_0, z)$  and  $(C''_0, z)$  are smooth and intersect with multiplicity  $m$ , then  $\delta_{\text{pl}}(C_0, z) = m - 1$ , the result derived from a thorough study of the versal deformation of singularity  $A_{2m-1}$ . We suggest another point of view on the problem, based on the patchworking construction, and obtain that in general

$$\delta_{\text{pl}}(C_0, z) = \delta(C'_0, z) + \delta(C''_0, z) + m - \max\{r', r''\} ,$$

where  $m$  is the intersection number of  $C'_0$  (or  $C''_0$ ) with the line  $\Sigma'_0 \cap \Sigma''_0$ , and  $r'$ ,  $r''$  are the numbers of local branches of  $C'_0$ ,  $C''_0$ , respectively. If, in addition, the germs  $(C'_0, z)$ ,  $(C''_0, z)$  are semiquasihomogeneous, we prove that the pair  $(\Sigma_t, C_t)$  with  $t \neq 0$  is homeomorphic to a pair  $(\mathbb{C}^2, A)$ ,  $A$  being an affine algebraic or pseudoholomorphic curve with a Newton polygon determined by the Newton diagrams of the given singularities  $(C'_0, z)$ ,  $(C''_0, z)$ .

We discuss some other examples as well.

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DIRK SIERSMA  
UTRECHT

CURVATURE AND THE GAUSS-BONNET DEFECT OF COMPLEX AFFINE  
HYPERSURFACES

We study the evolution of the curvature in families of affine hypersurfaces. Besides the local loss of curvature studied notably by Langevin, a new phenomenon occurs: the asymptotic loss of curvature towards infinity. We obtain formulas in terms of global polar invariants and we show that in certain cases, the vanishing curvature can be expressed in terms of local invariants of the compactified hypersurface. (joint work with Mihai Tibar)

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ANGELA MARIA SITTA  
IBILCE-UNESP

A NOTE ON THE NON-DEGENERATE UMBILICS AND THE PATH  
FORMULATION FOR BIFURCATION PROBLEMS

The theory of parametrised contact-equivalence of Golubitsky-Schaeffer is very successful for the understanding and classification of the qualitative local behaviour of bifurcation diagrams and their perturbations. Path formulation is an alternative point of view that organises contact-equivalence by allowing to distinguish the singular behaviour due to the core of the bifurcation germ (when the parameters vanish) from the effects of the way parameters enter.

In this work we show how it can be used to classify and structure efficiently multiparameter bifurcation problems. In corank 1 many results are already known. Here we are interested in corank 2 problems. In particular, the non degenerate umbilics singularities are the generic cores in four situations: the general or gradient problems and the  $\mathbb{Z}_2$ -equivariant (general or gradient) problems where  $\mathbb{Z}_2$  acts on the second component of  $\mathbb{R}^2$  via  $\kappa(x, y) = (x, -y)$ . The universal unfolding of the umbilic singularities have an interesting 'Russian doll' type of structure of universal unfoldings in all those categories.

One advantage of our approach is that we can handle one, two, or more, parameter situations using the same framework. We can even consider some special parameter structure (for instance some internal hierarchy). In this

work we classify the generic bifurcations with 1, 2 or 3 parameters that occur in those cases. Some classification results are known with one bifurcation parameter, but the other are new.

We discuss some application to the bifurcation of a cylindrical panel under different loads structure. This problem has many natural parameters that provide concrete examples of our generic diagrams around the first interaction of the buckling modes.

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JAWAD SNOUSSI

THE NASH MODIFICATION AND HYPERPLANE SECTIONS ON SURFACES

For the study of singularities of germs at a point of complex analytic surfaces, two particular modifications may be considered: the blow-up of the point and the Nash modification. Both transformations have desingularization virtues. In fact, the surface can be desingularized after a finite iteration of normalized point blow-ups ([7], [1]) or normalized Nash modifications ([6]).

The domination relation between these two modifications is related to hyperplane sections and polar curves and their base points after one or another modification.

It is well known that the normalized blow-up of a point factors through the Nash modification if and only if the family of local (absolute) polar curves does not have a base point after the blow-up ([2], [6]). These base points correspond to the so called “exceptional tangents” of the surface at the blown-up point ([4]). They are completely characterized in the case of normal surfaces in [5]. For the case of hypersurfaces of  $\mathbb{C}^3$  with non-isolated singularities we refer to [3].

In this work, we give a necessary and sufficient condition for the normalized Nash modification to factor through the blow-up of a point.

We first characterize the base points of hyperplane sections after Nash modification. We prove that these base points are in one-to-one correspondence with the planar components of the tangent cone of the surface at the considered point.

Then we prove that the normalized Nash modification of the surface factors through the blow-up of a point if and only if the tangent cone of the surface at that point does not have any planar component.

In the last section, we use the characterizations of the base points of the polar curves after the blow-up of a point, given in [5], to prove that, in the

case of normal surfaces, the normalized blow-up of a point dominates the normalized Nash modification if and only if they are isomorphic.

## References

- [1] R. Bondil and D.T. Lê, *Résolution des singularités de surfaces par éclatements normalisés (multiplicité, multiplicité polaire, et singularités minimales)*, Trends in singularities, Trends in Math., Birkhäuser Verlag, 2002, pp. 31–81.
- [2] G. González-Sprinberg, *Résolution de Nash des points doubles rationnels*, Ann. Inst. Fourier **32** (1982), no. 2, 111–178.
- [3] D.T. Lê and B. Teissier, *Sur la gomtrie des surfaces complexes I. Tangentes exceptionnelles*, Amer. J. of Math. **101** (1979), 420–452.
- [4] ———, *Limites d’espaces tangents en gomtrie analytique*, Comment. Math. Helvetici **63** (1988), 540–578.
- [5] J. Snoussi, *Limites d’espaces tangents à une surface normale*, Comment. Math. Helv. **76** (2001), 61–88.
- [6] M. Spivakovsky, *Sandwiched singularities and desingularization of surfaces by normalized Nash transformations*, Ann. of Math. **131** (1990), 411–491.
- [7] O. Zariski, *The reduction of singularities on an algebraic surface*, Ann. of Math. **40** (1939), 639–689.

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HUMBERTO SOARES

UNIVERSIDADE REGIONAL DO CARIRI

$C^\ell$ - $G$ -TRIVIALITY OF MAP GERMS AND NEWTON POLYHEDRA,  $G = \mathcal{R}$ ,  $\mathcal{C}$   
AND  $\mathcal{K}$

We provide a sufficient condition for the  $C^\ell$ - $G$ -triviality ( $G$  is one of Mather’s groups  $\mathcal{R}$ ,  $\mathcal{C}$  or  $\mathcal{K}$ ) of deformations of map germs  $f_t : (\mathbb{C}^n, 0) \rightarrow (\mathbb{C}^p, 0)$  of type  $f_t(x) = f(x) + th(x)$  which satisfy a Newton non-degeneracy condition. This condition is given in terms of the Newton filtration of the map germ  $h$ .

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ANDRAS SZUCS  
BORDISM GROUPS OF FOLD MAPS

Let us consider embeddings  $M^n \subset Q^q \times R^1$  such that the composition

$$M^n \subset Q^q \times R^1 \rightarrow Q^q$$

has only fold singularities. The bordism group of such embeddings will be computed. Moreover for each sequence of natural numbers

$$a_0, a_1, a_2, \dots$$

one can define the bordism group of those embeddings for which the projection has the property that the number of  $A_i$  type preimages is at most  $a_i$  for each point. We will show how to determine these groups.

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FARID TARI  
UNIVERSITY OF DURHAM  
ASYMPTOTIC, CHARACTERISTIC AND PRINCIPAL CURVES ON A  
CROSS-CAP

The study of the differential geometry of the cross-cap was initiated by J.W. Bruce and J.A. West (1998) (see also West's Ph.D. thesis, 1995). Singularity theory is used there to obtain information about the flat geometry of the cross-cap. The aim of our work here is to continue this investigation and study some natural pairs of foliations on the cross-cap.

Given an oriented smooth surface embedded in  $\mathbb{R}^3$ , there are natural pairs of foliations that capture its geometry. The well known foliations are the lines of principal curvature, the asymptotic and characteristic curves. The lines of curvature are those along which the normal curvature are extreme. They are defined everywhere on the surface and form an orthogonal net except at umbilic points where every direction can be considered as principal. Their configurations at umbilic were drawn by Darboux, but a rigorous proof is given in [Sotomayor-Gutierrez, 1982] and [Bruce-Fidal, 1986]. The global properties of these foliations are also studied in [Sotomayor-Gutierrez, 1982].

Asymptotic curves are those along which the normal curvature vanishes. They are defined in the closure of the hyperbolic region of the surface. They form a family of cusps at a generic parabolic point. Their configurations at cusps of Gauss are drawn in the book [Banchoff *etal*, 1982] and a more general approach is given in [Davydov, 1985]. Global properties of these foliations including the study of cycles are given in [Garcia-Sotomayor, 1997].

Characteristic directions are defined in the closure of the elliptic region. At elliptic points there is a unique pair of conjugate directions for which the included angle (i.e the angle between these directions) is minimal. These directions are called characteristic directions and their integral curves are called the characteristic curves. Their study is carried out in [Bruce-Tari, 2002] and [Garcia-Sotomayor, 2003]. In [Garcia-Sotomayor, 2003] they are labeled *harmonic mean curvature lines* and are defined as curves along which the normal curvature is  $K/H$ , where  $K$  is the Gauss curvature and  $H$  is the mean curvature.

The principal and asymptotic directions can be interpreted using singularity theory (folding maps and contact with lines). Interpreting the characteristic directions from the singularity theory viewpoint is still to be done.

In this work, we obtained the local configurations of the lines of curvature, the asymptotic and characteristic curves around a cross-cap. The results are obtained by studying binary differential equations (also known as quadratic differential equations) whose discriminant has a degenerate singularity. The key ingredient is an extended version of the blowing up technique used by Guíñez. We observe that the configurations of the lines of curvature at a cross-cap are obtained previously by [Garcia-Gutierrez-Stomayor, 2000]. We recover here the same results regarding the lines of curvature using an alternative approach.

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RENATO VIDAL MARTINS  
UFMG

### TRIGONAL NON-GORENSTEIN CURVES

We answer some questions on trigonal non-Gorenstein curves such as the number of non-Gorenstein points, the kind of such singularities, possible canonical models, uniqueness and number of base points of a pencil of degree 3, and the amplitude of the Maroni invariant.

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TERRY WALL  
TRANSVERSALITY OF FAMILIES OF MAPPINGS

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MINORU YAMAMOTO  
HOKKAIDO UNIVERSITY  
ON THE SPACE OF FOLD MAPS OF  $S^2$  TO  $R^2$  WITH A CONNECTED  
SINGULAR SET.

Let  $f : S^2 \rightarrow \mathbf{R}^2$  be a fold map such that the singular set of  $f$  is the equator of  $S^2$ . We denote by  $\mathcal{F}$  the set of all such fold maps. In 1970 and 1972, Eliashberg studied  $\mathcal{F}$  and he decided the homotopy type of this space. In this talk, we determine the connected components of  $\mathcal{F}$  by the elementary (combinatorial) method.

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MICHAEL ZHITOMIRSKII  
TECHNION, HAIFA  
GERMS AND MULTIGERMS OF INTEGRAL CURVES IN A CONTACT 3-SPACE

The talk is devoted to the theorem stating that two diffeomorphic germs or multigerms of integral curves in a contact 3-space are contactomorphic. At first I will explain why this theorem does not hold in contact spaces of bigger dimension. Then I will give a proof in the holomorphic category and will explain why the real-analytic case is much more difficult - it requires certain results, of independent significance, on distinguishing integral multigerms defining an orientation and integral multigerms admitting an orientation-reversing symmetry. The last part of the talk will be devoted to applications - classification of singular integral curves in a contact 3-space and classification of non-singular integral curves in an Engel 4-manifold and its generalization - Cartan-Goursat  $n$ -manifold.

# 4. Posters

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JOÃO NIVALDO TOMAZELLA

UNIVERSIDADE FEDERAL DE SÃO CARLOS

DEFORMATIONS WITH CONSTANT MILNOR NUMBER  
AND MULTIPLICITY OF COMPLEX HYPERSURFACES

We investigate the constancy of the Milnor number of one parameter deformations of holomorphic germs of functions  $f : (\mathbb{C}^n, 0) \rightarrow (\mathbb{C}, 0)$  with isolated singularity, in terms of some Newton polyhedra associate to such germs.

When the Jacobian ideals  $J(f_t) = \langle \partial f_t / \partial x_1, \dots, \partial f_t / \partial x_n \rangle$  of a deformation  $f_t(x) = f(x) + \sum_{s=1}^{\ell} \delta_s(t) g_s(x)$  are non-degenerate on some fixed Newton polyhedron  $\Gamma_+$ , we show that this family have constant Milnor number for small values of  $t$ , if and only if all germs  $g_s$  have non-decreasing  $\Gamma$ -order with respect to  $f$ . As a consequence of these results we give a positive answer to Zariski's question for Milnor constant families satisfying a non-degeneracy condition on the Jacobian ideals.

This is a joint work with Marcelo José Saia.

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MARIA ELENICE RODRIGUES HERNANDES (ICMC-USP)

SOME RELATIONS BETWEEN LOCAL INVARIANTS OF PLANE CURVES

Let  $\mathcal{C}$  be an irreducible algebroid plane curve and  $\phi : \mathbb{C} \rightarrow \mathbb{C}^2$  a primitive parametrization of  $\mathcal{C}$ .

Many relations among invariants for plane curves appear in the literature (see, [Ber],[B-G]), for example the well known Milnor's formula ([Mil]) relates the Milnor number with the delta invariant and the number of branches of the curve. Similarly, David Mond ([Mond]) has a formula relating these invariants and the called the image Milnor number.

In this work we establish a new formula relating the  $\mathcal{A}_e$ -codimension of the parametrization  $\phi$  and the classical invariants of plane curves.

## References

- [Ber] Berger, R.W., *Report on the torsion of the differential module of an algebraic curve*, Algebraic Geometry and its applications (1994), 285 - 303.
- [Mil] Milnor, J.W., *Singular points of complex hypersurfaces*, Princeton University Press, Princeton, 1968.
- [Mond] Mond, D., *Looking at bent wires -  $\mathcal{A}_e$ -codimension and the vanishing topology of parametrized curve singularities*, Math. Proc. Camb. Phil. Soc. 117 (1995), 213- 222.
- [B-G] Buchweitz, R.-O., Greuel, G.-M., *The Milnor number and deformations of complex curve singularities*, Inventiones Math. 58 (1980), 241-281.

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JOÃO CARLOS FERREIRA COSTA (ICMC-USP)

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ELIANA PINHO (UNIVERSIDADE DO PORTO, PORTUGAL)

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LIZANDRO SANCHEZ CHALLAPA

INDICES OF BINARY DIFFERENTIAL EQUATIONS

Supervisors: M. A. S. Ruas and F. Tari

In a smooth generic surface the lines of principal curvature provide two families of curves, with isolated singularities at the umbilic points. The index in this case is defined as the index determined by one family of these curves. In this work we define the index of the binary differential equations (BDE), at points at which the function discriminant has Morse singularities.