Exponentials in Ludics: How and at what price

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Not Separable I

$$\mathfrak{D} := \begin{array}{c} \maltese \\ (-, \xi i, J) \\ (+, \xi, I) \end{array} \qquad \mathfrak{D}' := \begin{array}{c} \maltese \\ (-, \xi i, J) \\ (+, \xi, I) \\ (-, \xi i, J) \\ (+, \xi, I) \end{array}$$

Not Separable II

Consider the following class of designs $\mathfrak{D}_{\mathsf{A,B}}$:

✠	A	B	✠
$(-, \xi.0, \{1\})$	$(-, \xi.0, \{2\})$	$(-, \xi.0, \{1\})$	$(-, \xi.0, \{2\})$
$(+, \xi, \{0\})$		$(+, \xi, \{0\})$	
$(-, \xi.0, \{1\})$		$(-, \xi.0, \{2\})$	
$(+, \xi, \{0\})$			

Separable I

$$\mathfrak{E} := \begin{array}{c} \maltese \\ (-, \xi.\bar{i}_{b}.i, J) \\ (+, \xi.\bar{i}_{b}, I) \end{array} \qquad \mathfrak{E}' := \begin{array}{c} \maltese \\ (-, \xi.\bar{i}_{r}.i, J) \\ (+, \xi.\bar{i}_{r}, I) \\ (-, \xi.\bar{i}_{b}.i, J) \\ (+, \xi.\bar{i}_{b}, I) \end{array}$$

Base: $\vdash \xi.\bar{i}$

Separable II

$$\maltese \qquad\qquad A \qquad\qquad B \qquad\qquad \maltese$$

$$(-,\xi.\bar{i}_r.0,\{1\}) \quad (-,\xi.\bar{i}_r.0,\{2\}) \quad (-,\xi.\bar{i}_g.0,\{1\}) \quad (-,\xi.\bar{i}_g.0,\{2\})$$

$$(+,\xi.\bar{i}_r,\{0\}) \qquad\qquad (+,\xi.\bar{i}_g,\{0\})$$

$$(-,\xi.\bar{i}_b.0,\{1\}) \qquad\qquad (-,\xi.\bar{i}_b.0,\{2\})$$

$$(+,\xi.\bar{i}_b,\{0\})$$

Base: $\vdash \xi.\bar{i}$

Remark

Observe pointers:

$$\maltese \qquad\qquad (+, \xi.\bar{i}_r.0.1, \{3\})$$

$$(-, \xi.\bar{i}_r.0, \{0\}) \qquad (-, \xi.\bar{i}_r.0, \{1\})$$

$$(+, \xi.\bar{i}_r, \{0\})$$

$$(-, \xi.\bar{i}_b.0, \{1\})$$

$$(+, \xi.\bar{i}_b, \{0\})$$

Base: $\vdash \xi.\bar{i}$

Remark

Observe pointers:

$$\maltese \qquad (+, \xi.\bar{i}_b.0.1, \{3\})$$

$$(-, \xi.\bar{i}_r.0, \{0\}) \qquad (-, \xi.\bar{i}_r.0, \{1\})$$

$$(+, \xi.\bar{i}_r, \{0\})$$

$$(-, \xi.\bar{i}_b.0, \{1\})$$

$$(+, \xi.\bar{i}_b, \{0\})$$

Base: $\vdash \xi.\bar{i}$

Example I

$$\mathfrak{F} := \begin{matrix} (+, \xi.\bar{i}_b.i, J) & (+, \xi.\bar{i}_r.j, J) \\ (-, \xi.\bar{i}_b, I) & (-, \xi.\bar{i}_r, I) \end{matrix}$$

Base: $\xi.\bar{i} \vdash$

Example II (Additives)

$$\mathfrak{F} := \begin{array}{ll} (+, \sigma.\bar{i}_{b}, L) & (+, \sigma.\bar{i}_{b}, K) \\ (-, \xi.\bar{j}_{r}, I) & (-, \xi.\bar{j}_{r}, J) \end{array}$$

Base: $\xi.\bar{i} \vdash \sigma.\bar{j}$

Normalization I : Termination

For $\mathfrak{E}$ design on base $\vdash \xi.\bar{i}$ and $\mathfrak{F}$ on $\xi.\bar{i} \vdash$ we have:

$\maltese$ $(-, \xi.\bar{i}_b.i, J)$ $(+, \xi.\bar{i}_b, I)$	$(+, \xi.\bar{i}_b.i, J)$ $(-, \xi.\bar{i}_b, I)$	$(+, \xi.\bar{i}_r.j, J)$ $(-, \xi.\bar{i}_r, I)$

Normalization I : Termination

For $\mathfrak{E}$ design on base $\vdash \xi.\bar{i}$ and $\mathfrak{F}$ on $\xi.\bar{i} \vdash$ we have:

$\maltese$ $(-, \xi.\bar{i}_b.i, J)$ $(+, \xi.\bar{i}_b, I)$	$(+, \xi.\bar{i}_b.i, J)$ $(-, \xi.\bar{i}_b, I)$	$(+, \xi.\bar{i}_r.j, J)$ $(-, \xi.\bar{i}_r, I)$

Normalization I : Termination

For $\mathfrak{E}$ design on base $\vdash \xi.\bar{i}$ and $\mathfrak{F}$ on $\xi.\bar{i} \vdash$ we have:

$\maltese$		
$(-,\xi.\bar{i}_b.i,J)$	$(+,\xi.\bar{i}_b.i,J)$	$(+,\xi.\bar{i}_r.j,J)$
$(+,\xi.\bar{i}_b,I)$	$(-,\xi.\bar{i}_b,I)$	$(-,\xi.\bar{i}_r,I)$

Normalization I : Termination

For $\mathfrak{E}$ design on base $\vdash \xi.\bar{i}$ and $\mathfrak{F}$ on $\xi.\bar{i} \vdash$ we have:

✠ $(-, \xi.\bar{i}_b.i, J)$ $(+, \xi.\bar{i}_b, I)$	$(+, \xi.\bar{i}_b.i, J)$ $(-, \xi.\bar{i}_b, I)$	$(+, \xi.\bar{i}_r.j, J)$ $(-, \xi.\bar{i}_r, I)$

Normalization I : Termination

For $\mathfrak{E}$ design on base $\vdash \xi.\bar{i}$ and $\mathfrak{F}$ on $\xi.\bar{i} \vdash$ we have:

$\mathfrak{E}$	$\mathfrak{F}$	
$\maltese$		
$(-, \xi.\bar{i}_b.i, J)$	$(+, \xi.\bar{i}_b.i, J)$	$(+, \xi.\bar{i}_r.j, J)$
$(+, \xi.\bar{i}_b, I)$	$(-, \xi.\bar{i}_b, I)$	$(-, \xi.\bar{i}_r, I)$

Normalization I : Termination

For $\mathfrak{E}$ design on base $\vdash \xi.\bar{i}$ and $\mathfrak{F}$ on $\xi.\bar{i} \vdash$ we have:

$$[\![\mathfrak{E}, \mathfrak{F}]\!] = \mathfrak{Dai}$$

on base $\vdash$ (*closed* net).

Normalization II : Failure

For $\mathfrak{E}'$ design on base $\vdash \xi.\bar{i}$ and $\mathfrak{F}$ on $\xi.\bar{i} \vdash$ we have:

$\maltese$		
$(-, \xi.\bar{i}_r.i, J)$	$(+, \xi.\bar{i}_b.i, J)$	$(+, \xi.\bar{i}_r.j, J)$
$(+, \xi.\bar{i}_r, I)$	$(-, \xi.\bar{i}_b, I)$	$(-, \xi.\bar{i}_r, I)$
$(-, \xi.\bar{i}_b.i, J)$		
$(+, \xi.\bar{i}_b, I)$		

Normalization II : Failure

For $\mathfrak{E}'$ design on base $\vdash \xi.\bar{i}$ and $\mathfrak{F}$ on $\xi.\bar{i} \vdash$ we have:

$\maltese$		
$(-, \xi.\bar{i}_r.i, J)$	$(+, \xi.\bar{i}_b.i, J)$	$(+, \xi.\bar{i}_r.j, J)$
$(+, \xi.\bar{i}_r, I)$	$(-, \xi.\bar{i}_b, I)$	$(-, \xi.\bar{i}_r, I)$
$(-, \xi.\bar{i}_b.i, J)$		
$(+, \xi.\bar{i}_b, I)$		

Normalization II : Failure

For $\mathfrak{E}'$ design on base $\vdash \xi.\bar{i}$ and $\mathfrak{F}$ on $\xi.\bar{i} \vdash$ we have:

✠		
$(-, \xi.\bar{i}_r.i, J)$		
$(+, \xi.\bar{i}_r, I)$	$(+, \xi.\bar{i}_b.i, J)$	$(+, \xi.\bar{i}_r.j, J)$
$(-, \xi.\bar{i}_b.i, J)$	$(-, \xi.\bar{i}_b, I)$	$(-, \xi.\bar{i}_r, I)$
$(+, \xi.\bar{i}_b, I)$		

Normalization II : Failure

For $\mathfrak{E}'$ design on base $\vdash \xi.\bar{i}$ and $\mathfrak{F}$ on $\xi.\bar{i} \vdash$ we have:

✠		
$(-, \xi.\bar{i}_r.i, J)$		
$(+, \xi.\bar{i}_r, I)$	$(+, \xi.\bar{i}_b.i, J)$	$(+, \xi.\bar{i}_r.j, J)$
$(-, \xi.\bar{i}_b.i, J)$	$(-, \xi.\bar{i}_b, I)$	$(-, \xi.\bar{i}_r, I)$
$(+, \xi.\bar{i}_b, I)$		

Normalization II : Failure

For $\mathfrak{E}'$ design on base $\vdash \xi.\bar{i}$ and $\mathfrak{F}$ on $\xi.\bar{i} \vdash$ we have:

$\maltese$		
$(-, \xi.\bar{i}_r.i, J)$	$(+, \xi.\bar{i}_b.i, J)$	$(+, \xi.\bar{i}_r.j, J)$
$(+, \xi.\bar{i}_r, I)$	$(-, \xi.\bar{i}_b, I)$	$(-, \xi.\bar{i}_r, I)$
$(-, \xi.\bar{i}_b.i, J)$		
$(+, \xi.\bar{i}_b, I)$		

Normalization II : Failure

For $\mathfrak{E}'$ design on base $\vdash \xi.\bar{i}$ and $\mathfrak{F}$ on $\xi.\bar{i} \vdash$ we have:

✠		
$(-, \xi.\bar{i}_r.i, J)$	$(+, \xi.\bar{i}_b.i, J)$	$(+, \xi.\bar{i}_r.j, J)$
$(+, \xi.\bar{i}_r, I)$	$(-, \xi.\bar{i}_b, I)$	$(-, \xi.\bar{i}_r, I)$
$(-, \xi.\bar{i}_b.i, J)$		
$(+, \xi.\bar{i}_b, I)$		

Normalization II : Failure

For $\mathfrak{E}'$ design on base $\vdash \xi.\bar{i}$ and $\mathfrak{F}$ on $\xi.\bar{i} \vdash$ we have:

✠		
$(-, \xi.\bar{i}_r.i, J)$	$(+, \xi.\bar{i}_b.i, J)$	$(+, \xi.\bar{i}_r.j, J)$
$(+, \xi.\bar{i}_r, I)$	$(-, \xi.\bar{i}_b, I)$	$(-, \xi.\bar{i}_r, I)$
$(-, \xi.\bar{i}_b.i, J)$		
$(+, \xi.\bar{i}_b, I)$		

Normalization II : Failure

For $\mathfrak{E}'$ design on base $\vdash \xi.\bar{i}$ and $\mathfrak{F}$ on $\xi.\bar{i} \vdash$ we have:

$\mathfrak{E}'$	$\mathfrak{F}$	
$\maltese$		
$(-, \xi.\bar{i}_r.i, J)$	$(+, \xi.\bar{i}_b.i, J)$	$(+, \xi.\bar{i}_r.j, J)$
$(+, \xi.\bar{i}_r, I)$	$(-, \xi.\bar{i}_b, I)$	$(-, \xi.\bar{i}_r, I)$
$(-, \xi.\bar{i}_b.i, J)$		
$(+, \xi.\bar{i}_b, I)$		

Normalization II : Failure

For $\mathfrak{E}'$ design on base $\vdash \xi.\bar{i}$ and $\mathfrak{F}$ on $\xi.\bar{i} \vdash$ we have:

$$[\![\mathfrak{E}', \mathfrak{F}]\!] = \Omega$$

on base $\vdash$

Example

Let $\mathfrak{D}, \mathfrak{E}$ be designs on base respectively $\xi.\bar{i} \vdash \beta.\bar{j}$ and $\vdash \xi.\bar{i}, \sigma$.

$\mathfrak{D}$		$\mathfrak{E}$
	$(+, \xi.\bar{i}_b.j, N)$	$(+, \xi.\bar{i}_r, J)$
	$(-, \beta.\bar{j}_g.k, M)$	$(-, \xi.\bar{i}_b.j, N)$
$(+, \beta.\bar{j}_p, K)$	$(+, \beta.\bar{j}_g, K)$	$(+, \xi.\bar{i}_b, J)$
$(-, \xi.\bar{i}_r, J)$	$(-, \xi.\bar{i}_b, J)$	$(-, \sigma.k, R)$
		$(+, \sigma, K)$

Example

Let $\mathfrak{D}, \mathfrak{E}$ be designs on base respectively $\xi.\bar{i} \vdash \beta.\bar{j}$ and $\vdash \xi.\bar{i}, \sigma$.

$$(+, \xi.\bar{i}_b.j, N)$$
$$(-, \beta.\bar{j}_g.k, M)$$
$$(+, \beta.\bar{j}_p, K) \qquad (+, \beta.\bar{j}_g, K)$$
$$(-, \xi.\bar{i}_r, J) \qquad (-, \xi.\bar{i}_b, J)$$

$$(+, \xi.\bar{i}_r, J)$$
$$(-, \xi.\bar{i}_b.j, N)$$
$$(+, \xi.\bar{i}_b, J)$$
$$(-, \sigma.k, R)$$
$$(+, \sigma, K)$$

Example

Let $\mathfrak{D}, \mathfrak{E}$ be designs on base respectively $\xi.\bar{i} \vdash \beta.\bar{j}$ and $\vdash \xi.\bar{i}, \sigma$.

$\mathfrak{D}$		$\mathfrak{E}$
	$(+, \xi.\bar{i}_b.j, N)$	$(+, \xi.\bar{i}_r, J)$
	$(-, \beta.\bar{j}_g.k, M)$	$(-, \xi.\bar{i}_b.j, N)$
$(+, \beta.\bar{j}_p, K)$	$(+, \beta.\bar{j}_g, K)$	$(+, \xi.\bar{i}_b, J)$
$(-, \xi.\bar{i}_r, J)$	$(-, \xi.\bar{i}_b, J)$	$(-, \sigma.k, R)$
		$(+, \sigma, K)$

Example

Let $\mathfrak{D}, \mathfrak{E}$ be designs on base respectively $\xi.\bar{i} \vdash \beta.\bar{j}$ and $\vdash \xi.\bar{i}, \sigma$.

	$(+, \xi.\bar{i}_b.j, N)$	$(+, \xi.\bar{i}_r, J)$
	$(-, \beta.\bar{j}_g.k, M)$	$(-, \xi.\bar{i}_b.j, N)$
$(+, \beta.\bar{j}_p, K)$	$(+, \beta.\bar{j}_g, K)$	$(+, \xi.\bar{i}_b, J)$
$(-, \xi.\bar{i}_r, J)$	$(-, \xi.\bar{i}_b, J)$	$(-, \sigma.k, R)$
		$(+, \sigma, K)$

Example

Let $\mathfrak{D}, \mathfrak{E}$ be designs on base respectively $\xi.\bar{i} \vdash \beta.\bar{j}$ and $\vdash \xi.\bar{i}, \sigma$.

	$(+, \xi.\bar{i}_b.j, N)$	$(+, \xi.\bar{i}_r, J)$
	$(-, \beta.\bar{j}_g.k, M)$	$(-, \xi.\bar{i}_b.j, N)$
$(+, \beta.\bar{j}_p, K)$	$(+, \beta.\bar{j}_g, K)$	$(+, \xi.\bar{i}_b, J)$
$(-, \xi.\bar{i}_r, J)$	$(-, \xi.\bar{i}_b, J)$	$(-, \sigma.k, R)$
		$(+, \sigma, K)$

Example

Let $\mathfrak{D}, \mathfrak{E}$ be designs on base respectively $\xi.\bar{i} \vdash \beta.\bar{j}$ and $\vdash \xi.\bar{i}, \sigma$.

	$(+, \xi.\bar{i}_b.j, N)$	$(+, \xi.\bar{i}_r, J)$
	$(-, \beta.\bar{j}_g.k, M)$	$(-, \xi.\bar{i}_b.j, N)$
$(+, \beta.\bar{j}_p, K)$	$(+, \beta.\bar{j}_g, K)$	$(+, \xi.\bar{i}_b, J)$
$(-, \xi.\bar{i}_r, J)$	$(-, \xi.\bar{i}_b, J)$	$(-, \sigma.k, R)$
		$(+, \sigma, K)$

Example

Let $\mathfrak{D}, \mathfrak{E}$ be designs on base respectively $\xi.\bar{i} \vdash \beta.\bar{j}$ and $\vdash \xi.\bar{i}, \sigma$.

	$(+, \xi.\bar{i}_b.j, N)$	$(+, \xi.\bar{i}_r, J)$
	$(-, \beta.\bar{j}_g.k, M)$	$(-, \xi.\bar{i}_b.j, N)$
$(+, \beta.\bar{j}_p, K)$	$(+, \beta.\bar{j}_g, K)$	$(+, \xi.\bar{i}_b, J)$
$(-, \xi.\bar{i}_r, J)$	$(-, \xi.\bar{i}_b, J)$	$(-, \sigma.k, R)$
		$(+, \sigma, K)$

Example

Let $\mathfrak{D}, \mathfrak{E}$ be designs on base respectively $\xi.\bar{i} \vdash \beta.\bar{j}$ and $\vdash \xi.\bar{i}, \sigma$.

$$
\llbracket \mathfrak{D}, \mathfrak{E} \rrbracket := \quad \begin{array}{c} (+, \beta.\bar{j}_p, K) \\ (-, \beta.\bar{j}_g.k, M) \\ (+, \beta.\bar{j}_g, K) \\ (-, \sigma.k, R) \\ (+, \sigma, K) \end{array}
$$

on base $\vdash \beta.\bar{j}, \sigma$ (*open* net).

About separation...

Let $\mathfrak{D}_{\mathrm{A,B}}$ and $\mathfrak{E}$ be designs on base respectively $\vdash \xi.\bar{i}$ and $\xi.\bar{i} \vdash$

$$\maltese \qquad \mathrm{A} \qquad \mathrm{B} \qquad \maltese$$

$$(-, \xi.\bar{i}_r.0, \{1\}) \quad (-, \xi.\bar{i}_r.0, \{2\}) \quad (-, \xi.\bar{i}_g.0, \{1\}) \quad (-, \xi.\bar{i}_g.0, \{2\})$$

$$(+, \xi.\bar{i}_r, \{0\}) \qquad (+, \xi.\bar{i}_g, \{0\})$$

$$(-, \xi.\bar{i}_b.0, \{1\}) \qquad (-, \xi.\bar{i}_b.0, \{2\})$$

$$(+, \xi.\bar{i}_b, \{0\})$$

$$(+, \xi.\bar{i}_b.0, \{1\}) \quad (+, \xi.\bar{i}_r.0, \{2\})$$

$$(-, \xi.\bar{i}_b, \{0\}) \quad (-, \xi.\bar{i}_r, \{0\})$$

About separation...

Let $\mathfrak{D}_{\text{A,B}}$ and $\mathfrak{E}$ be designs on base respectively $\vdash \xi.\bar{i}$ and $\xi.\bar{i} \vdash$

$$\maltese \qquad \text{A} \qquad \text{B} \qquad \maltese$$

$$(-, \xi.\bar{i}_r.0, \{1\}) \quad (-, \xi.\bar{i}_r.0, \{2\}) \quad (-, \xi.\bar{i}_g.0, \{1\}) \quad (-, \xi.\bar{i}_g.0, \{2\})$$

$$(+, \xi.\bar{i}_r, \{0\}) \qquad (+, \xi.\bar{i}_g, \{0\})$$

$$(-, \xi.\bar{i}_b.0, \{1\}) \qquad (-, \xi.\bar{i}_b.0, \{2\})$$

$$(+, \xi.\bar{i}_b, \{0\})$$

$$(+, \xi.\bar{i}_b.0, \{1\}) \quad (+, \xi.\bar{i}_r.0, \{2\})$$

$$(-, \xi.\bar{i}_b, \{0\}) \quad (-, \xi.\bar{i}_r, \{0\})$$

About separation...

Let $\mathfrak{D}_{\mathrm{A,B}}$ and $\mathfrak{E}$ be designs on base respectively $\vdash \xi.\bar{i}$ and $\xi.\bar{i} \vdash$

$$\dfrac{\dfrac{\dfrac{\dfrac{\maltese}{(-,\xi.\bar{i}_r.0,\{1\})} \quad \dfrac{\mathrm{A}}{(-,\xi.\bar{i}_r.0,\{2\})}}{(+,\xi.\bar{i}_r,\{0\})}}{(-,\xi.\bar{i}_b.0,\{1\})} \quad \dfrac{\dfrac{\dfrac{\mathrm{B}}{(-,\xi.\bar{i}_g.0,\{1\})} \quad \dfrac{\maltese}{(-,\xi.\bar{i}_g.0,\{2\})}}{(+,\xi.\bar{i}_g,\{0\})}}{(-,\xi.\bar{i}_b.0,\{2\})}}{(+,\xi.\bar{i}_b,\{0\})}$$

$$\dfrac{(+,\xi.\bar{i}_b.0,\{1\})}{(-,\xi.\bar{i}_b,\{0\})} \qquad \dfrac{(+,\xi.\bar{i}_r.0,\{2\})}{(-,\xi.\bar{i}_r,\{0\})}$$

About separation...

Let $\mathfrak{D}_{A,B}$ and $\mathfrak{E}$ be designs on base respectively $\vdash \xi.\bar{i}$ and $\xi.\bar{i} \vdash$

$$\maltese \qquad A \qquad B \qquad \maltese$$

$$(-, \xi.\bar{i}_r.0, \{1\}) \quad (-, \xi.\bar{i}_r.0, \{2\}) \quad (-, \xi.\bar{i}_g.0, \{1\}) \quad (-, \xi.\bar{i}_g.0, \{2\})$$

$$(+, \xi.\bar{i}_r, \{0\}) \qquad (+, \xi.\bar{i}_g, \{0\})$$

$$(-, \xi.\bar{i}_b.0, \{1\}) \qquad (-, \xi.\bar{i}_b.0, \{2\})$$

$$(+, \xi.\bar{i}_b, \{0\})$$

$$(+, \xi.\bar{i}_b.0, \{1\}) \qquad (+, \xi.\bar{i}_r.0, \{2\})$$

$$(-, \xi.\bar{i}_b, \{0\}) \qquad (-, \xi.\bar{i}_r, \{0\})$$

About separation...

Let $\mathfrak{D}_{A,B}$ and $\mathfrak{E}$ be designs on base respectively $\vdash \xi.\bar{i}$ and $\xi.\bar{i} \vdash$

$$\maltese \qquad A \qquad B \qquad \maltese$$

$$(-, \xi.\bar{i}_r.0, \{1\}) \quad (-, \xi.\bar{i}_r.0, \{2\}) \quad (-, \xi.\bar{i}_g.0, \{1\}) \quad (-, \xi.\bar{i}_g.0, \{2\})$$

$$(+, \xi.\bar{i}_r, \{0\}) \qquad (+, \xi.\bar{i}_g, \{0\})$$

$$(-, \xi.\bar{i}_b.0, \{1\}) \qquad (-, \xi.\bar{i}_b.0, \{2\})$$

$$(+, \xi.\bar{i}_b, \{0\})$$

$$(+, \xi.\bar{i}_b.0, \{1\}) \quad (+, \xi.\bar{i}_r.0, \{2\})$$

$$(-, \xi.\bar{i}_b, \{0\}) \quad (-, \xi.\bar{i}_r, \{0\})$$

About separation...

Let $\mathfrak{D}_{A,B}$ and $\mathfrak{E}$ be designs on base respectively $\vdash \xi.\bar{i}$ and $\xi.\bar{i} \vdash$

$$\maltese \qquad A \qquad B \qquad \maltese$$

$$(-, \xi.\bar{i}_r.0, \{1\}) \quad (-, \xi.\bar{i}_r.0, \{2\}) \quad (-, \xi.\bar{i}_g.0, \{1\}) \quad (-, \xi.\bar{i}_g.0, \{2\})$$

$$(+, \xi.\bar{i}_r, \{0\}) \qquad (+, \xi.\bar{i}_g, \{0\})$$

$$(-, \xi.\bar{i}_b.0, \{1\}) \qquad (-, \xi.\bar{i}_b.0, \{2\})$$

$$(+, \xi.\bar{i}_b, \{0\})$$

$$(+, \xi.\bar{i}_b.0, \{1\}) \qquad (+, \xi.\bar{i}_r.0, \{2\})$$

$$(-, \xi.\bar{i}_b, \{0\}) \qquad (-, \xi.\bar{i}_r, \{0\})$$

About separation...

Let $\mathfrak{D}_{A,B}$ and $\mathfrak{E}$ be designs on base respectively $\vdash \xi.\bar{i}$ and $\xi.\bar{i} \vdash$

$$\maltese \qquad A \qquad B \qquad \maltese$$

$$(-, \xi.\bar{i}_r.0, \{1\}) \quad (-, \xi.\bar{i}_r.0, \{2\}) \quad (-, \xi.\bar{i}_g.0, \{1\}) \quad (-, \xi.\bar{i}_g.0, \{2\})$$

$$(+, \xi.\bar{i}_r, \{0\}) \qquad (+, \xi.\bar{i}_g, \{0\})$$

$$(-, \xi.\bar{i}_b.0, \{1\}) \qquad (-, \xi.\bar{i}_b.0, \{2\})$$

$$(+, \xi.\bar{i}_b, \{0\})$$

$$(+, \xi.\bar{i}_b.0, \{1\}) \qquad (+, \xi.\bar{i}_r.0, \{2\})$$

$$(-, \xi.\bar{i}_b, \{0\}) \qquad (-, \xi.\bar{i}_r, \{0\})$$

About separation...

Let $\mathfrak{D}_{A,B}$ and $\mathfrak{E}$ be designs on base respectively $\vdash \xi.\bar{i}$ and $\xi.\bar{i} \vdash$

$$\maltese \qquad A \qquad B \qquad \maltese$$

$$(-, \xi.\bar{i}_r.0, \{1\}) \quad (-, \xi.\bar{i}_r.0, \{2\}) \quad (-, \xi.\bar{i}_g.0, \{1\}) \quad (-, \xi.\bar{i}_g.0, \{2\})$$

$$(+, \xi.\bar{i}_r, \{0\}) \qquad (+, \xi.\bar{i}_g, \{0\})$$

$$(-, \xi.\bar{i}_b.0, \{1\}) \qquad (-, \xi.\bar{i}_b.0, \{2\})$$

$$(+, \xi.\bar{i}_b, \{0\})$$

$$(+, \xi.\bar{i}_b.0, \{1\}) \qquad (+, \xi.\bar{i}_r.0, \{2\})$$

$$(-, \xi.\bar{i}_b, \{0\}) \qquad (-, \xi.\bar{i}_r, \{0\})$$

About separation...

Let $\mathfrak{D}_{\mathsf{A,B}}$ and $\mathfrak{E}$ be designs on base respectively $\vdash \xi.\bar{i}$ and $\xi.\bar{i} \vdash$

$$\maltese \qquad \mathsf{A} \qquad \mathsf{B} \qquad \maltese$$

$$(-, \xi.\bar{i}_r.0, \{1\}) \quad (-, \xi.\bar{i}_r.0, \{2\}) \quad (-, \xi.\bar{i}_g.0, \{1\}) \quad (-, \xi.\bar{i}_g.0, \{2\})$$

$$(+, \xi.\bar{i}_r, \{0\}) \qquad (+, \xi.\bar{i}_g, \{0\})$$

$$(-, \xi.\bar{i}_b.0, \{1\}) \qquad (-, \xi.\bar{i}_b.0, \{2\})$$

$$(+, \xi.\bar{i}_b, \{0\})$$

$$(+, \xi.\bar{i}_b.0, \{1\}) \quad (+, \xi.\bar{i}_r.0, \{2\})$$

$$(-, \xi.\bar{i}_b, \{0\}) \quad (-, \xi.\bar{i}_r, \{0\})$$

Behaviours (Types)

A set of designs **G** closed by bi-orthogonal ($\mathbf{G} = \sim\sim \mathbf{G}$).

In general, given a set of design **H** :

$$\sim \mathbf{H} := \{\mathfrak{E} : \forall \mathfrak{D} \in \mathbf{H}, [\![\mathfrak{D}, \mathfrak{E}]\!] = \maltese\}$$

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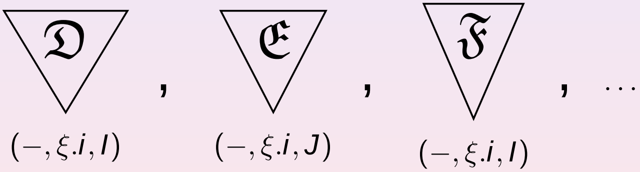
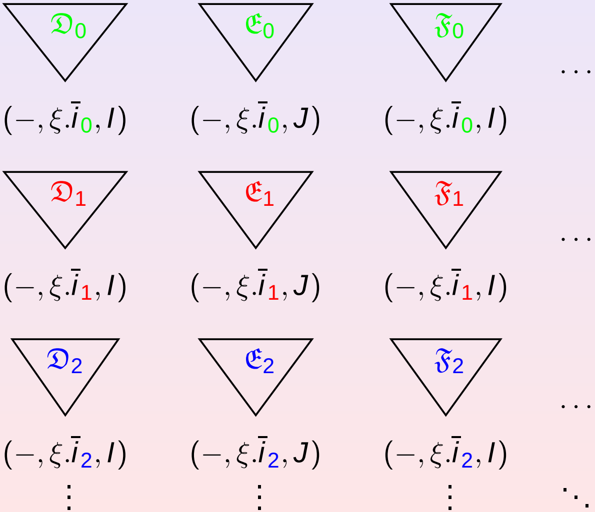
Exponentials I : $\sharp$

1)Let **N** be a behaviour on base $\xi.i \vdash$ and let us consider its designs :



Exponentials II : $\sharp$

2) Consider now the following set of designs $\mathbf{N}^{\mathbb{N}}$:



Exponentials III : $\sharp$

In general, $\mathfrak{D}_a$ is obtained by substituting each occurrence of i.address $\xi.i.\alpha$ in actions of $\mathfrak{D}$ by $\xi.\bar{i}_a.\alpha$.
For example, let us consider:

$$\mathfrak{D} = \dfrac{\dfrac{\dfrac{\maltese}{(-, \xi.i.j.k', L)} \qquad \dfrac{\maltese}{(-, \xi.i.j.k'', M)}}{(+, \xi.i.j, K)}}{(-, \xi.i, J)}$$

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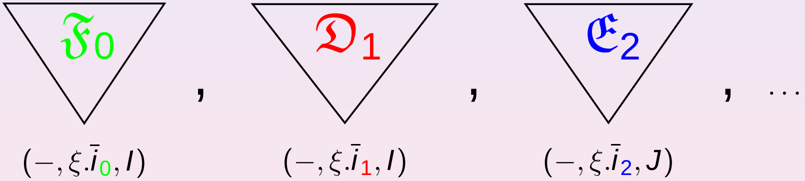
Exponentials IV : $\sharp$

3) Consider now any "formal diagonal" of $\mathbf{N}^{\mathbb{N}}$, for example :



Exponentials V : $\sharp$

4) Finally, let us consider the union of such designs:

$$\mathfrak{F}_0 \quad \cup \quad \mathfrak{D}_1 \quad \cup \quad \mathfrak{E}_2 \quad \cup \quad \dots$$

$$(-, \xi.\bar{i}_0, I) \qquad (-, \xi.\bar{i}_1, I) \qquad (-, \xi.\bar{i}_2, J)$$

which is a design of $\sharp\mathbf{N}$ on base $\xi.\bar{i} \vdash$

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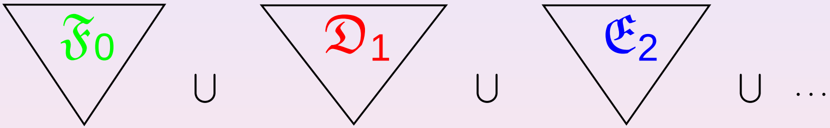

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Uniformity

We observe that designs such as:

$$\mathfrak{F}_0 \cup \mathfrak{D}_1 \cup \mathfrak{E}_2 \cup \ldots$$

are non uniform , whereas uniform designs have form:

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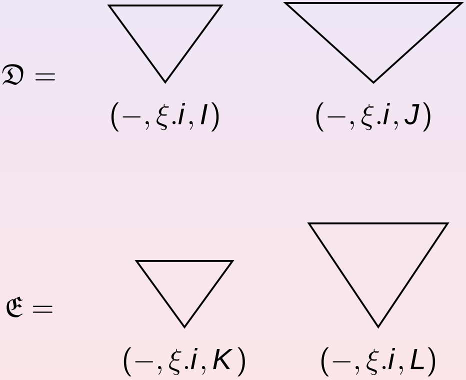
Exponentials VI : $\flat$

We define

$$\flat \mathbf{P} := \sim \sharp \sim \mathbf{P}$$

Exponential isomorphism I

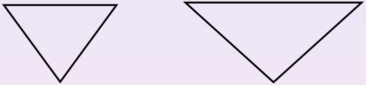
Consider $\mathfrak{D} \in \mathbf{N}$ and $\mathfrak{E} \in \mathbf{M}$ on the same base $\xi.i \vdash$ such that the sets of their first action are disjoint.



Exponential isomorphism II

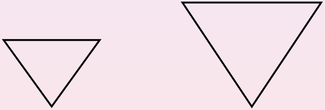
Consider $\mathfrak{D}' \in \sharp\mathbf{N}$ and $\mathfrak{E}' \in \sharp\mathbf{M}$ on the same base $\xi.\bar{i} \vdash$:

$\mathfrak{D}' =$



$(-, \xi.\bar{i}_0, I) \qquad (-, \xi.\bar{i}_0, J) \qquad \ldots$

$\mathfrak{E}' =$

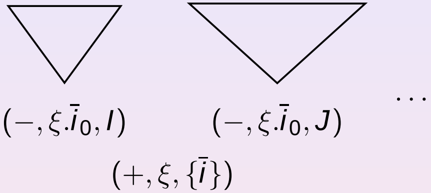


$(-, \xi.\bar{i}_0, K) \qquad (-, \xi.\bar{i}_0, L) \qquad \ldots$

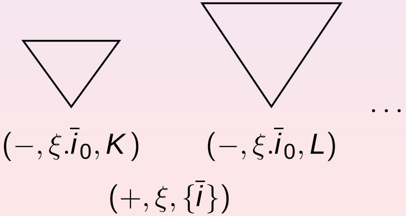
Exponential isomorphism III

Consider $\mathfrak{D}'' \in$!**N** and $\mathfrak{E}'' \in$!**M** on the same base $\xi \vdash$:

$\mathfrak{D}'' =$

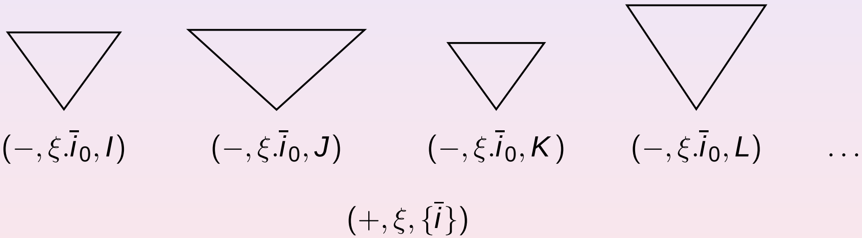


$\mathfrak{E}'' =$



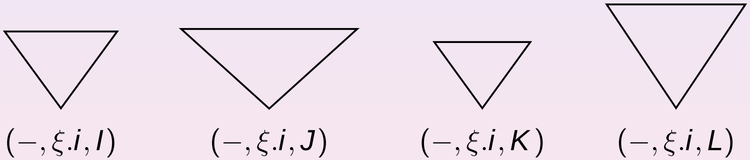
Exponential isomorphism IV

Finally, consider $\mathfrak{D}'' \otimes \mathfrak{E}'' \in\ !\mathbf{N} \otimes !\mathbf{M}$ on base $\xi \vdash$:



Exponential isomorphism V

Conversely, consider $\mathfrak{F} \in \mathbf{N} \,\&\, \mathbf{M}$ on base $\xi.i \vdash$



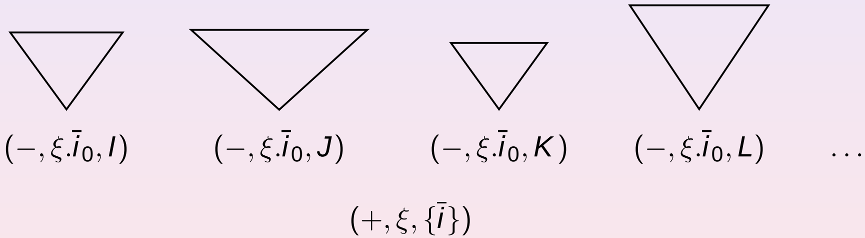
Exponential isomorphism VI

Consider $\mathfrak{F}' \in \sharp(\mathbf{N} \mathbin{\&} \mathbf{M})$ on base $\xi.\bar{i} \vdash$

$(-, \xi.\bar{i}_0, I)$ $(-, \xi.\bar{i}_0, J)$ $(-, \xi.\bar{i}_0, K)$ $(-, \xi.\bar{i}_0, L)$ $\ldots$

Exponential isomorphism VII

And finally $\mathfrak{F}'' \in !(\mathbf{N} \mathbin{\&} \mathbf{M})$ on base $\xi \vdash$



In **MALL**$_{foc}$:

Theorem (Soundness)

To every proof π *of a sequent* $\vdash \Gamma$ *in* **MALL**$_{foc}$ *we can associate a design* $\pi^* \in (\vdash \Gamma)^*$.

Theorem (Correctness)

$$\pi_1 \rightsquigarrow \pi_2 \quad \Rightarrow \quad \pi_1^* = \pi_2^*$$

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Full Completeness

Theorem (Full Completeness)

Let $\vdash \Gamma$ *be a sequent and let* $\mathfrak{D}$ *be a uniform design in its interpretation* $(\vdash \Gamma)^*$*, then we can find a cut free proof* π *in* $\mathbf{LL}_{foc}^{\uparrow\downarrow}$ *of* $\vdash \Gamma$ *such that* $\pi^* = \mathfrak{D}$*.*

Formulas of $\mathbf{LL}^{\uparrow\downarrow}_{foc}$

Decomposition of exponentials:

$$! = \downarrow\sharp \qquad ? = \uparrow\flat$$

Formulas:

$$P ::= \mathbf{0} \mid P \otimes P \mid P \oplus P \mid \downarrow N \mid !N$$

$$N ::= \top \mid N \parr N \mid N \,\&\, N \mid \uparrow P \mid ?P$$

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$\mathbf{LL}^{\uparrow\downarrow}_{foc}$

$$\frac{}{\vdash \Gamma, \top} \qquad \frac{\vdash \Gamma, P}{\vdash \Gamma, {\uparrow}P} \qquad \frac{\vdash \Gamma, N}{\vdash \Gamma, {\downarrow}N}$$

$$\frac{\vdash \Gamma, P_i}{\vdash \Gamma, P_1 \oplus P_2} \qquad \frac{\vdash \Gamma_1, P_1 \quad \vdash \Gamma_2, P_2}{\vdash \Gamma_1, \Gamma_2, P_1 \otimes P_2} \qquad \frac{\vdash N_1, N_2, \Gamma}{\vdash N_1 \parr N_2, \Gamma}$$

$$\frac{\vdash N_1, \Gamma \quad \vdash N_2, \Gamma}{\vdash N_1 \mathbin{\&} N_2, \Gamma}$$

$$\frac{\vdash \flat\Gamma, N}{\vdash \flat\Gamma, \sharp N} \qquad \frac{\vdash \Gamma, P}{\vdash \Gamma, \flat P} \qquad \frac{\vdash \Gamma, \flat P, \flat P}{\vdash \Gamma, \flat P}$$

$\mathbf{LL}_{foc}^{\uparrow\downarrow}$

$$\frac{}{\vdash \Gamma, \top} \qquad \frac{\vdash \Gamma, P}{\vdash \Gamma, \uparrow P} \qquad \frac{\vdash \Gamma, N}{\vdash \Gamma, \downarrow N}$$

$$\frac{\vdash \Gamma, P_i}{\vdash \Gamma, P_1 \oplus P_2} \qquad \frac{\vdash \Gamma_1, P_1 \quad \vdash \Gamma_2, P_2}{\vdash \Gamma_1, \Gamma_2, P_1 \otimes P_2} \qquad \frac{\vdash N_1, N_2, \Gamma}{\vdash N_1 \parr N_2, \Gamma}$$

$$\frac{\vdash N_1, \Gamma \quad \vdash N_2, \Gamma}{\vdash N_1 \,\&\, N_2, \Gamma}$$

$$\frac{\vdash \flat\Gamma, N}{\vdash \flat\Gamma, \sharp N} \qquad \frac{\vdash \Gamma, P}{\vdash \Gamma, \flat P} \qquad \frac{\vdash \Gamma, \flat P, \flat P}{\vdash \Gamma, \flat P}$$

Further Directions

- Ludics as a model of (Linear) π-calculus.
- Hyland-Ong style treatment of exponentials in Ludics framework.

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