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Figures of dialogue: a view from Ludics

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Abstract In this paper, we study dialogue as a game, but not only in the sense in which there would exist winning strategies and a priori rules. Dialogue is not governed by game rules like for chess or other games, since even if we start from a priori rules, it is always possible to play with them, provided that some invariant properties are preserved. An important discovery of Ludics is that such properties may be expressed in geometrical terms. The main feature of a dialogue is "convergence". Intuitively, a dialogue "diverges" when it stops prematurely by some disruption, or a violation of the tacit agreed upon conditions of the discourse. It converges when the two speakers go together towards a situation where they agree at least on some points. As we shall see, convergence may be thought of through the geometrical concept of *orthogonality*. Utterances in a dialogue have as their content, not only the processes (similar to proofs) which lead to them from a monologic view, but also their *interactions* with other utterances. Finally, any utterance must be seen as co-constructed in an interaction between two processes. That is to say that it not only contains one speaker's intentions but also his or her expectations from the other interlocutor. From our viewpoint, discursive strategies like *narration*, *elaboration*, *topicalization* may derive from such interactions, as well as speech acts like *assertion, question* and *denegation*.

Keywords Dialogue · Pragmatics · Logic · Game theory

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1 Introduction

1.1 From proofs to interaction

Traditionally, logic stands on two feet: model theory and proof theory. Of course, most of the time, there are completeness theorems which allow one to translate what is said in one language into the other, but it is often observed that one of the two languages is better suited for expressing some things rather than others. For instance, the field of semantics, which has been founded on the Fregean paradigm, may call for either model-theoretic or proof-theoretic concepts, but generally, it is rather atuned towards the first. Classically, meaning is thought of in model-theoretic, truth-conditional terms. Seminal works by Carnap, followed by Kripke and Hintikka, have added possible worlds into the picture, so that we are now familiar with the idea that the propositional content of a sentence is expressed by a set of possible worlds. In a more dynamic view, such a propositional content is seen as a Context Change Potential, that is a transition from set to set. Recently, Inquisitive Semantics (Groenendijk 2009; Groenendijk and Roelofsen 2010) has renewed these views, adapting them to the dialogue situation, making of the content of an utterance not simply a set of possible worlds, but a set of possibilities (that is maximal sets of such worlds supporting the claim), nevertheless, it rests on the same model-theoretic notions as well.

On the other hand, there is another approach, although much less dominant, which is based on *proof theory*, mainly illustrated by the work of Ranta (1994), itself based on works by Martin-Löf (1984).[1] According to Ranta, a text may be formalized by a sequence of judgements $x_i : A(x_1, \ldots, x_{i-1})$, where the x_i's are proofs of the statements $A(x_1, \ldots, x_{i-1})$ which, themselves are seen dependent of other proof-objects. This renders the progressive aspect of a text. The sequence of judgements on which an assertion is dependent is called its *context*. This conception is extended to treat questions, but these new objects require additional rules for interpretation, such as Ranta's "fairness" rule and "hearer" rule. To characterize a speech act, you must, in Ranta's words, explain what act is performed by means of an expression of that form, for instance:

You may make the assertion $\vdash A$ if you know a proof of A

This leaves room for other forms, for instance propositional questions $A \mid B$ may be introduced by giving a rule "stating how the **hearer** must react to it":

Answer the question $A \mid B$ by asserting either $\vdash A$ or $\vdash B$

This makes crucial reference to the hearer. If we take that option seriously, this entails that we can no longer content ourselves with a single *monologic* viewpoint as expressed by proof theory. Our main concern here is to complete this approach by adding *interaction* contexts to proof contexts. This intention forces us to alter the background slightly. Besides *proofs*, there are now *counter-proofs*. There are neither 'fairness rule" nor "hearer rule" explicitly given.

[1] See also Sundholm (1986).

1.2 From fallacies to pragmatics

If in a dialogue $\mathcal{D}$, some participant asks a question σ like "what is the square root of 27?", we shall say that a convergent dialogue takes place if the answerer can provide τ, an attempt to prove the existence of a number n such that $n^2 = 27$. In this case, the questioner may be seen as attempting to refute that there is such square root and the answerer as attempting to prove it. The dialogue ends up either by providing a proof of the claim that there is such square root, or by providing a refutation of it. Attempts of proofs and attempts of refutation interact until some result is produced. Convergence means that both the hearer and the fairness rules are satisfied. If not, a divergence occurs. This happens in case of presuppositional failure, for instance if there are several square roots. In this case, the interaction process is blocked, and there is no result at all. This occurs in Aristotle's well known "several questions in one" fallacy, such as when the judge asks the young man: "*Have you stopped beating your father?*" thereby forcing him to answer by "yes" or "no", although he may simply argue that he never beat his father. Our framework can take into account such a situation in a very precise way (Lecomte and Quatrini 2010).

In this paper, we will not only address the issue of questions and answers but also more general aspects of discourse and dialogue. The treatment of questions is paradigmatic in that it shows the need to take into account the two positions always present in a dialogue: the speaker and the hearer. But we may then generalize this observation by speculating that every speech act must be seen in the same way. For instance, an *assertion* prepares the kind of objections that a potential hearer could make and predicts what arguments the speaker may give in order to respond correctly. A *denegation*, as has been shown by Ducrot (1984), cannot be interpreted simply starting from the sentence it seems to denegate. When someone says "*she is not nice, on the contrary she is execrable*", he or she does not produce a negation of the sentence "*she is not nice*" but of the sentence it is supposed to contradict in a dialogue, that is another interlocutor who would sustain that "*she is nice*".

The game of discourse is a multi-participant game and the framework we propose is just a way to take that aspect seriously in consideration.[2]

1.3 From referentialist to inferentialist semantics

Another point which deserves attention is that in a proof-theoretic approach, propositions are not equated with their truth values but with the sets of their proofs. To make a judgement is therefore to exhibit a proof or at least to assert that we have such a proof. In the ludical setting we propose here (Girard 2001, 2003, 2006), this is slightly weakened: to make a judgement is simply to have justifications for one's claims, in response to potential objections, represented by *tests* (or counter-proofs) coming from the other speaker. We can easily envisage a situation in which our claim

[2] The idea that discourse is governed by underlying implicit questions has also previously been explored by van Kuppevelt (1996), but it was much less formalized than it is the case here.

is not sustained by an authentic proof, but is nevertheless able to pass a finite number of tests. It is, after all, the only requirement we face in ordinary conversations in our daily life.

Our approach is therefore compatible with the perspective according to which "inferential practices which include those of production and consumption of *reasons* are at the heart of linguistic activity". This is the approach taken by some philosophers, most notably (Brandom 2000). Consequently, it is compatible with the *inferentialist* conception of meaning rather than with the *referentialist* view.

Considering the completeness results mentioned above, what is striking for Contructive Type Theory (Martin-Löf 1984) as well as for Ludics is that *external* completeness is simply not relevant. Martin-Löf's Type Theory is presented as a framework which requires no external interpretion in terms of Set Theory since it encompasses Set Theory (propositions and sets are identified when propositions are interpreted as their sets of proofs). In the same way, Ludics could be interpreted by means of Game semantics but the Game theory that would be needed is already incorporated in Ludics, since the primitive objects (*designs*) may be viewed as *proofs* as well as *strategies* in a game. We then face a situation where the duality between proofs and (counter-) models may be gotten rid of, something well illustrated by Girard's aphorism according to which "the meaning of the rules lies in the rules themselves".

In Sect. 2, we give an overview of the spirit of the paper. Speech acts are seen as two-sided processes. We try to present dialogues as games, but not in the usual sense which implies gain functions and winning strategies. Rather, the principle of dialogues is not *to win* but *to keep the dialogue convergent as long as possible*. Section 3 brings elements of formalization, presenting the main concepts of Ludics. Section 4 shows how these concepts may be used in order to build a real theory of dialogues. Some elements of the Ludics framework (*loci*) are shown to be interpretable as *questions under discussion*. This perspective enables us to treat dialogues as global processes as well as ongoing incremental processes which proceed step by step. We thus present opposing *dialogue frames* and the trace of their interaction, regarded as a Dialogue Gameboard in the sense of Ginzburg et al. (2009). Section 5 addresses the topic of discursive relations and Sect. 6 provides our conclusion.

2 Interaction: between what things?

2.1 Dialogue as interaction between two processes

2.1.1 Positive and negative actions

Let us suppose we have two *processes*, that is sequences of alternate steps, some positive and others negative. These processes may be viewed as two informational flows like the sequences of actions and reactions coming from two speakers in a dialogue. The positive steps represent positive actions, such as asserting something, or giving something, while the latter represent negative actions or passive attitudes, like recording something, accepting something or planning a range of alternative answers to some question.[3] We may envisage that the positive steps of one process coincide with negative steps of the other, in the sense that, what is recorded or accepted in the passive attitude is precisely what is asserted or given in the positive action of the other side. Of course, it would be perfect if such exchanges continue until some state of felicity is reached, but events may occur which could disrupt this harmony. In fact, felicity is a big word…we may simply expect that one of the two processes agrees to stop when it has obtained everything that it wanted to obtain from the other. This can be "all the reasons" that a speaker wished to obtain from the interlocutor, or all the types of behaviour that are needed in order to achieve a complex task. When all these elements have been obtained, one of the processes could stop and say "ok, now we can stop communicating, even if temporarily". This type of process is applicable in many fields.

2.1.2 The case of proof searching

Mathematics, for instance, illustrates a straightforward interpretation of such a confrontation between two processes. We may suppose that one process is an attempt of finding a proof for some proposition, and the other is an attempt to provide refutations of the arguments proposed in the first. Here, we can see the situation as a classical opposition with a winner and a looser. The winning process is of course the one which succeeds. There are actually two main results: in one, the first process "wins", that is, the refutation-seeker fails in his attempts and can simply give up. In the other possibility, it is the second process that "wins"—an objection was raised that the first side did not respond to, providing a counter-example for the proposition which was supposed to be proven. There are other possible results: the interaction could stop for reasons other than the two above, or the list of counter-arguments turns out to be infinite (or circular). In the latter case, the interaction may continue infinitely.

2.1.3 The case of ordinary language

In ordinary language, *stopping* a process through such positive action does not necessarily signal giving the win to the contradictor, but may consist of a simple *acknowledgement*, a way of expressing, for instance, that the answer given by the hearer has been accepted and acknowledged. When the interaction stops before such a situation is reached, it is mainly because the set of answers (or reactions) offered by the active speaker does not include the potential reaction of the hearer. For instance, in the fallacy example, the potential answer "*I never beat my father*" is not a part of the set of options offered by the first speaker, a set which consists only of {"*yes*", "*no*"}. This underlines the important role played by the negative actions. Actually, we assume that there is a time to ask or say something and just after, a time to plan the other speaker's reactions. A positive action that is not predicted leads to a disruption. Of course, by responding and actively asserting something, the other speaker enters the same process: she also has the potential reactions of the first interlocutor in mind (perhaps new questions),

[3] Answering a question, responding to a request, but also acknowledging are of course not negative actions, but positive ones.

she therefore makes a negative step which displays the potential reactions of the first speaker.

2.2 Keeping dialogue convergent

These considerations seem very simple, and in fact are. However, despite their straightforwardness, they enable us to understand some fundamental features of a conversation. In usual Game Theory, the rationality is governed by the objective of maximizing one's gain, the main goal is to reach a winning situation, and emphasis is put on *winning* strategies. In Ludics, winning is *secondary*, replaced by another goal: *keeping the dialogue converging*. There are of course games in which players' primary aim is winning (argumentative debates for instance), but most "games" are not of this type. We mentioned above the collaboration between two searchers: one looking for a proof and the other trying to find counter-arguments. We can assume that both participants are interested in the same result: determining whether a proposition is true or false, rather than having opposing goals, knowing who is right in the debate. We may interpret Grice's Cooperative Principle exactly in that spirit. In Grice's example:

A: *My car broke down*
B: *There is a garage just round the corner*

B's reaction may be interpreted only on the basis of a principle which states that *dialogue must be as convergent as possible*. We translate this principle through the fact that A, on his subsequent negative step, postulates a set of possible answers coming from B, all of which imply a way to solve his problem, and that B's reaction belongs to that set.

It follows from this principle that every speech turn involves two intricate dimensions, the speaker and the hearer. Speech acts are therefore *two-sided processes*, and it is this aspect which makes figures of dialogue and discourse what they are, as shown below.

In the next section, we give a presentation of the tools and concepts of Ludics that we will use in developing a theory of dialogue. Such a framework will be relevant to provide a precise definition of:

– the interaction between two processes,
– a notion of convergence between two processes,
– a notion of strategy in a dialogue.

3 Ludics as a Game Theory

Ludics is typically presented starting from the main concepts of Linear Logic, a logic which may be polarized, that is, such that its rules have polarities, either $+$ or $-$, thus making proofs sequences of polarized steps. Results coming from Theoretical Computer Science (Andréoli 1992) lead us to focalized proofs, that is proofs as alternating sequences of steps. Games also give us examples of sequences of steps of opposite polarities. In this paper, we shall start from the Games perspective.

In Ludics, the concept of proof is subsumed under the concept of *design*, which may be seen either as a proof search or as a *strategy*.

As a strategy a design is a set of plays (**chronicles**) ending with the answers of the player to the moves of his opponent. The plays themselves are alternating sequences of moves (**actions**). Each move is given by three types of data: a **polarity** according to which, one player's view being fixed, this player's moves are positive, while those of the opponent are negative, a **focus**, that is the location (*locus*) of the move, a **ramification**, which represents the finite set of locations which can be reached in one step. A special positive move is provided by the so-called **daïmon**: no location is reached from it, since it provides a way of terminating a chronicle.

Locations are addresses or **loci**, coded by finite sequences of integers and sometimes denoted by small Greek letters. A locus which has a sequence σ as its initial subsequence is said to be a sublocus of the locus σ.

The starting positions or **forks** are denoted by $\Gamma \vdash \Delta$, where Γ and Δ are finite sets of loci, with Γ empty or reduced to only one locus. If Γ contains one locus, plays start from it and begin by an opponent's move (and the fork is said to be negative), otherwise the player starts from one of the *loci* in Δ (and the fork is said positive).

3.1 The basic definitions

The ludical notions are defined by Girard (2001) as follows:

Definition 1 (*Action, Chronicles*)

- An **action** κ is:
  - either a positive proper action $(+, \xi, I)$ or a negative proper action $(-, \xi, I)$ where the *focus* ξ of the action is a locus, and the *ramification* I is a finite set of integers,
  - or the positive action daimon denoted by †.
- A **chronicle** $\mathfrak{c}$ is a non-empty and finite alternate sequence of actions such that:
  - a positive proper action is either justified, i.e. its focus is built by one of the previous actions in the sequence, or it is called initial.
  - a negative action may be initial, in such a case it is the first action of the chronicle. Otherwise it is justified by the immediate previous positive action.
  - actions have distinct focuses.
  - if present, a daimon ends the chronicle.

  The **base** of the chronicle is a sequent $\Gamma \vdash \Delta$ of loci where the set Γ contains the initial focus of a negative action, if there is one and Δ contains the initial foci of future positive actions.
- Two chronicles $\mathfrak{c}_1$ and $\mathfrak{c}_2$ are **coherent**, when the two following conditions are satisfied:
  - either one extends the other or they first differ on negative actions, i.e., if $\mathfrak{c}_1 = w\kappa_1$ and $\mathfrak{c}_2 = w\kappa_2$ are coherent then either $\kappa_1 = \kappa_2$ or κ_1 and κ_2 are negative actions.
  - when they first differ on negative actions and these negative actions have distinct foci then the foci of following actions in $\mathfrak{c}_1$ and $\mathfrak{c}_2$ are pairwise distinct,

i.e. (where w is a chronicle or an empty sequence of actions) $w(-, \xi_1, I_1)\sigma_1$ and $w(-, \xi_2, I_2)\sigma_2$ are coherent with $\xi_1 \neq \xi_2$ iff σ_1 and σ_2 have distinct foci.

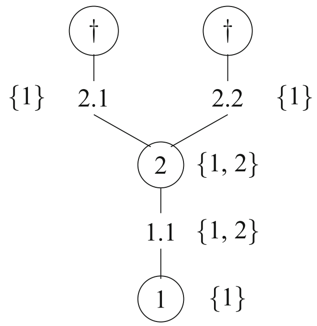
Definition 2 (*Designs, Nets*)

- A **design** $\mathfrak{D}$, based on $\Gamma \vdash \Delta$, is a set of chronicles based on $\Gamma \vdash \Delta$, such that the following conditions are satisfied:
  - The set is prefix closed.
  - The set is a clique of chronicles (the chronicles are pairwise coherent).
  - A chronicle without extension in $\mathfrak{D}$ ends with a positive action.
  - $\mathfrak{D}$ is non-empty when the base is positive (in that case all the chronicles begin with a unique positive action).
- A **net** is a finite set of designs on disjoint bases.

Example 1 Let us consider the two following chronicles:

$$\mathfrak{c}_1 = (+, 1, \{1\})\ (-, 1.1, \{1, 2\})\ (+, 2, \{1, 2\})\ (-2.1, \{1\})\ \dagger$$
$$\mathfrak{c}_2 = (+, 1, \{1\})\ (-, 1.1, \{1, 2\})\ (+, 2, \{1, 2\})\ (-2.2, \{1\})\ \dagger$$

The set of the prefixes of $\mathfrak{c}_1$ and $\mathfrak{c}_2$ is a design. It may be represented by a tree[4] (particular case of a forest), called *tree of actions*, where circles denote positive actions, nodes without circles negative ones, the list of integers inside each node is the selected focus for the action denoted and the set annexed to each node is a ramification:



3.2 Interaction

Generalizing the cut elimination procedure of the proof-theoretical perspective, interaction is a normalization of particular nets of designs, called *cut-nets* (Girard 2001). In such nets, addresses in bases are either all distinct or present only once in a negative position of a base and once in a positive one of another base, hence creating pairs which define *cuts*. The graph made of bases as vertices and cuts as links must be acyclic and connected. We give below the definition of interaction in the case of a *closed* cut-net, that is a net where all addresses in bases are parts of some cut. In this

[4] This presentation was introduced by Faggian (2002).

case, a *main design* may be distinguished: it has a positive base. The reader may find in Girard (2001) the definition for the general case.

Definition 3 (*Interaction on closed cut-nets*) Let $\mathfrak{R}$ be a closed cut-net, the design resulting from the interaction, denoted by $[[\mathfrak{R}]]$, is defined in the following way: let $\mathfrak{D}$ be the main design of $\mathfrak{R}$, with first action κ,

- if κ is a daimon, then $[[\mathfrak{R}]] = \{\dagger\}$,
- otherwise κ is a proper positive action $(+, \sigma, I)$ such that σ is part of a cut with another design. Let $\mathcal{N}$ be the set of all ramifications of (negative) actions on the same focus σ in this design):
  - If $I \notin \mathcal{N}$, then interaction fails.
  - Otherwise, interaction follows with the connected part of subdesigns obtained from I with the rest of $\mathfrak{R}$.

Following this definition, either interaction fails, or it does not terminate, or it results in the design $\mathcal{D}ai^{+} = \{\dagger\}$.

Definition 4 (*Orthogonality*) Two designs $\mathfrak{D}$ and $\mathfrak{E}$ respectively based on $\vdash \xi$ and $\xi \vdash$, are said orthogonal when $[[\mathfrak{D}, \mathfrak{E}]] = \{\dagger\}$. $\mathfrak{D}^{\perp}$ denotes the set of all designs $\mathfrak{E}$ such that $\mathfrak{D}$ and $\mathfrak{E}$ are orthogonal.

It is then possible to compare two designs according to their counter-designs. Moreover the separation theorem (Girard 2001) ensures that a design is exactly defined by its orthogonal: if $\mathfrak{D}^{\perp} = \mathfrak{E}^{\perp}$ then $\mathfrak{D} = \mathfrak{E}$.

Example 2 Let $\mathfrak{D}$ be the design defined by the set of prefixes of the following chronicles:

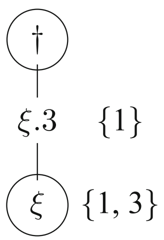
$$(-, \xi, \{1, 2\}), (+, \dagger)$$
$$(-, \xi, \{1, 3\}), (+, \xi.3, \{1\})$$

and $\mathfrak{E}$ by:

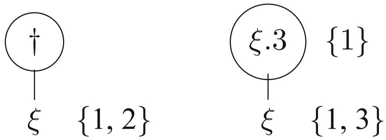
$$(+, \xi, \{1, 3\}), (-, \xi.3, \{1\}), (+, \dagger)$$

They may be represented by the following trees of actions:

- $\mathfrak{E}$:



– $\mathfrak{D}$:



Actually, the interaction between designs is made more intuitive by using representations of designs as trees of actions, since, in this case, normalization simply corresponds to a travel which starts from the first positive node of the main design, at each proper positive action, moves to the corresponding negative one (if there is one, if not, normalization fails), moves upward to the unique action which follows, and successfully terminates on the special positive action †, as shown in the following figure.

We are more interested in the *trace* of the interaction than in the normalized form.[5] Intuitively, as evident in the previous example, this simply consists of the sequence of actions which are "visited" during the normalization process.

Definition 5 (*The trace of an interaction*) Let $(\mathfrak{D}, \mathfrak{R})$ be a closed cut-net such that the normalisation process stops on convergence or divergence after a finite number n of steps. The trace of interaction, seen from the $\mathfrak{D}$'s view is denoted by $\tau_{\mathfrak{D}}$. We define by induction on $1 \leq k \leq n$ the initial subsequences of $\tau_{\mathfrak{D}}$:

- Case $k = 1$:
  - If the interaction stops in one step: either $\mathfrak{D}$ is equal to $\mathcal{D}ai^+$, in this case $\tau_{\mathfrak{D}} = \dagger$, or the main design (which is not $\mathfrak{D}$) is equal to $\mathcal{D}ai^+$ and in this case $\tau_{\mathfrak{D}}$ is the empty sequence, or the interaction fails and $\tau_{\mathfrak{D}}$ is put equal to Ω, the symbol used for *divergence*.[6]
  - Otherwise let κ^+ be the first action of the main design. The first action of $\tau_{\mathfrak{D}}$ is κ^+ if $\mathfrak{D}$ is the main design and is $\overline{\kappa^+}$ otherwise.[7]

[5] In the closed case, when the interaction converges, there is only one result: $\mathcal{D}ai^+ = \{\dagger\}$.

[6] Which is of course not an action properly speaking, but just a convenience.

[7] Where $\overline{\kappa}$ is defined such that $\overline{(+, \xi, I)} = (-, \xi, I)$, $\overline{(-, \xi, I)} = (+, \xi, I)$, an operation which can be be extended to sequences by $\overline{\epsilon} = \epsilon$ and $\overline{w\kappa} = \overline{w}\,\overline{\kappa}$.

- Case $k = p + 1$: the prefix $\kappa_1 \ldots \kappa_p$ of $\tau_{\mathfrak{D}}$ is already defined.
  - Either the interaction stops and:
    - if it converges, then $\tau_{\mathfrak{D}} = \kappa_1 \ldots \kappa_p$ if the main design is a subdesign of $\mathfrak{R}$, or $\tau_{\mathfrak{D}} = \kappa_1 \ldots \kappa_p \dagger$ if the main design is a subdesign of $\mathfrak{D}$.
    - if it fails, $\tau_{\mathfrak{D}} = \kappa_1 \ldots \kappa_p \Omega$.
  - or the interaction does not stop. Let κ^+ be the first proper action of the closed cut-net obtained after the step p, $\tau_{\mathfrak{D}}$ begins with $\kappa_1 \ldots \kappa_p \kappa^+$ if the main design is a subdesign of $\mathfrak{D}$ otherwise $\tau_{\mathfrak{D}}$ begins with $\kappa_1 \ldots \kappa_p \overline{\kappa^+}$.

REBRAND : By construction, the trace of an interaction is an alternating finite sequence of actions. All these actions are proper except the last one. The last action may be proper or is either equal to † or to Ω.

3.3 A proof-like presentation of designs

3.3.1 Designs as proofs

Girard (2001) provides a proof-like presentation of designs, called "designs as *dessins*".[8] This presentation is not faithful since several *dessins* may be associated with the same design. Nevertheless, for our purpose, the notions coincide and we will use the proof-like presentation below. The rules used to build a design as "dessin" may be understood as logical rules in an *hypersequentialized* calculus, that is rules with an arbitrary number of premisses. A negative rule is reversible (like are the & and ⅋ right rules in linear logic (Girard 1987)), which means that no information is lost when passing from the bottom to the top. A positive rule is not reversible, similarly to the ⊗ and ⊕ right rules in linear logic. Irreversible choices are made when using them.

Definition 6

- A **design** (as *dessin*) based on a fork $\Gamma \vdash \Delta$ is a tree of forks built by means of the three following rules:

  - DAÏMON

$$\frac{}{\vdash \Delta}\dagger$$

  - POSITIVE RULE

$$\frac{\ldots \xi.i \vdash \Delta_i \ldots}{\vdash \Delta, \xi}(+, \xi, I)$$

    where I is a ramification, i.e. a finite set of integers (maybe empty) ; for $i \in I$, the Δ_i's are pairwise disjoint and included in Δ.

  - NEGATIVE RULE

$$\frac{\ldots \vdash \xi.I, \Delta_I \ldots}{\xi \vdash \Delta}(-, \xi, \mathfrak{N})$$

    $\mathfrak{N}$ is a set (maybe empty or infinite) of ramifications, i.e. a set of finite sets of integers. For all $I \in \mathfrak{N}$, the Δ_I's, not necessarily disjoint, are contained in Δ.

[8] Thus contrasting with "designs as *desseins*".

Notations: If $I = \{i_1, i_2, \ldots, i_n, \ldots\}$, $\xi.I$ is the sequence $\xi.i_1, \xi.i_2, \ldots, \xi.i_n, \ldots$.

REBRAND : The case when the union of the Δ_i's is strictly contained in Δ (and similarly for the negative rule, when Δ_I is strictly contained in Δ) corresponds to the weakening rule in Logic.

Example 3 The designs $\mathfrak{D}$ and $\mathfrak{E}$ seen in Example 2 may be represented by

$$\dfrac{\dfrac{}{\vdash \xi.1, \xi.2}\dagger \qquad \dfrac{\xi.3.1 \vdash \xi.1}{\vdash \xi.1, \xi.3}(+, \xi.1, \{1\})}{\xi \vdash}(-, \xi, \{\{1, 2\}, \{1, 3\}\}) \qquad\qquad \dfrac{\xi.1 \vdash \qquad \dfrac{\dfrac{}{\vdash \xi.3.1}\dagger}{\xi.3 \vdash}(-, \xi.3, \{1\})}{\vdash \xi}(+, \xi, \{1, 3\})$$

3.3.2 Infinite processes: the example of $\mathcal{F}ax$

The following example provides us with an infinite process, which will be used in Sect. 5. The design is recursively defined.

$$Fax_{\xi,\xi'} = \dfrac{\dfrac{\dfrac{\ldots \quad Fax_{\xi_{i1},\xi'_{i1}} \quad \ldots}{\ldots \xi'.i \vdash \xi.i \ldots}}{\ldots \quad \vdash \xi.J_1, \xi' \quad \ldots}(+, \xi', J_1)}{\xi \vdash \xi'}(-, \xi, \mathcal{P}_f(\mathbb{N}))$$

It is called $\mathcal{F}ax$ because the result of an interaction between a design $\mathfrak{D}$ based on $\vdash \xi$ and $\mathcal{F}ax$ based on $\xi \vdash \xi'$ results in the copy of $\mathfrak{D}$ on the base $\vdash \xi'$.

3.4 Games in Ludics

By "games", we will understand more abstract objects than the usual games of dialogical logic (Lorenzen 1960; Lorenz 1961). In the latter games, each player's move is determined by previous moves of the other player and consists of a simple *rule* to apply. We may say that players play rules, while at a more abstract level, we may think of players playing *designs*, as if they adapted their strategies throughout their game. What they aim to do is mainly *to keep their strategies convergent*. This view seems to be more in accordance with the notion of *language games* as developed by philosophers (Wittgenstein 1953; Pietarinen 2007).[9] This view is therefore based on the concept of *orthogonality*. We may imagine that each player has a set of designs. A dialogue game arises when the sets are in duality with regard to each other.

[9] While many authors have tried to identify Wittgenstein's notion of "language game" and the notion of game which occurs in Game Theoretical Semantics, other authors have pointed out that there cannot be an identification of both notions. Hodges for instance (quoted in Pietarinen (2007)) argues that Wittgenstein's language games are such that "no one ever wins or loses such games".

Definition 7 (*Game*) A *game* is defined by a pair $(\mathbf{G}, \mathbf{G}^{\perp})$ of sets of designs such that:

$$(\forall \mathfrak{D})\ (\forall \mathfrak{E}) \quad \mathfrak{D} \in \mathbf{G} \wedge \mathfrak{E} \in \mathbf{G}^{\perp} \Rightarrow \mathfrak{D} \perp \mathfrak{E}$$

Let us call I and YOU the two participants in a dialogue. If I is identified with **G** and YOU with $\mathbf{G}^{\perp}$, it is assumed that every time I plays some $\mathfrak{D} \in \mathbf{G}$, YOU plays some $\mathfrak{E} \in \mathbf{G}^{\perp}$.

As noted by Girard (2006), in such a situation, one set may be considered the game rule for the other. Of course every change in one set entails a corresponding change in the other.

4 Dialogue from the proof perspective

4.1 Ludics and theory of dialogue

Dialogue has many features in common with the way in which Ludics intends to reconstruct Logic, on an interaction basis. The analysis of a dialogue amounts, at first, to highlighting the interaction process of which it is a trace. Ludics provides us with a framework in which most concepts can be interpreted in the dialogue perspective, thus opening the field for a new and real theory of dialogue. For instance, before they are closed by some intervention of the other speaker materialized by the action daïmon, sequences of loci that originate from the base of a positive design are good candidates for representing what Ginzburg et al. (2009) calls *questions under discussion* (in short: *QUD*).[10] The positive and the negative rules may also be interpreted in that direction, as will be illustrated in the following examples. We may postulate that, in a theory of dialogue, such as in Ludics, our aim is:

– to show that every intervention consists of opening new loci, some of which are focalized by the other speaker, and
– to associate positive and negative actions on a same locus, showing they correspond to each other.

In this paper, we concentrate on the formal aspect of dialogue, that is the organization of *loci*, and their dynamics, which results in the normalization procedure we presented in the previous section. The content of *loci* is another story and seems to be captured by more classical means—syntax and semantics. Nevertheless, it is our position that ludical tools can still be useful in recovering the contents, as tentatively shown in Lecomte and Quatrini (2010).

[10] Ginzburg et al. define a QUD as "a question that constitutes a "live issue", that is a question that has been introduced for discussion at a given point in the conversation and whose discussion has not yet been concluded".

4.2 Dialogues from a strictly interactive viewpoint

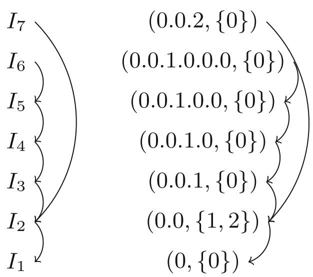
Let us consider a short example, taken from the novel "The Count of Monte-Cristo" by Alexandre Dumas. This is a dialogue between two characters, the Abbé Faria (F) and Edmond Dantès (E), where F tries to help E finding who could benefit from his death.

F	(*Who could benefit from your death?*) What was your life at this time?	I_1
E	I was ready to become captain of the *Pharaon*; I was about to marry a beautiful young girl.	I_2
F	Was anyone interested in you not becoming the captain of the *Pharaon*?	I_3
E	[. . .], Only one man. [. . .]	I_4
F	Who was he?	I_5
E	Danglars.	I_6
F	Well, tell me about that young girl…	I_7

Assume we are interested in linking interventions of both speakers, showing in what respect some of them provide *justifications* to others. We say that one intervention justifies another when it creates the *question* on which the second is anchored. In the foregoing dialogue, all the interventions, except the first and the last are justified by the immediately preceding ones, while the first one is initial and the last one is justified by the second intervention. For example, I_3 is justified by I_2, since the intervention I_2 introduces the propositional content according to which *Edmond was going to become captain of the Pharaon*. I_7 is also justified by I_2 because the fact *Edmond was about to marry* is introduced in I_2. The relation of justification between $I_1, \ldots, I_7$ is schematized on the left hand side of Fig. 1, while, on the right hand side of the same figure, justification is expressed in ludical terms, i.e. in pairs (*locus*, *ramification*). In this way, we represent the dialogue as a structured set of locations: in a pair (ξ, I), the locus ξ indicates the location from which a proposition is assumed to be anchored and the ramification I indicates the new *loci* that this proposition creates, i.e. the loci from which subsequent propositions will take place. Notice that such a ramification is not absolutely determined by the proposition but is just a possibility among several ones. It is the continuation of the dialogue which makes this more precise. In this ludical approach, propositions need not be fixed a priori, they are fixed afterwards, after the process has been carried out.

We may then observe that the sequence of justified interventions may be seen as the trace of an interaction. For that, we only have to add polarities to the pairs (ξ, I),

Fig. 1 An alternating sequence of justified interventions



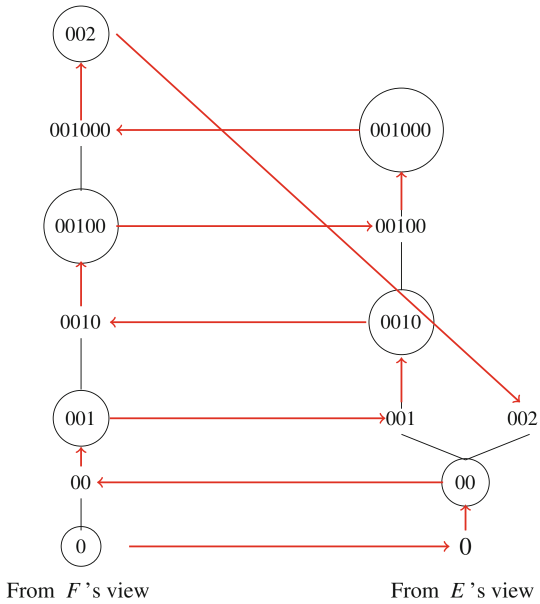


Fig. 2 The dialogue between Edmond and Faria seen as an interaction between two designs

according to the speaker who utters the content. The sequence of alternating actions thus obtained may be seen as the trace of the interaction between two designs which represent the dialogue as seen from each speaker's point of view. Figure 2 shows the dialogue between Edmond and Faria as an interaction between two designs.[11]

In this figure, we choose to represent the interacting designs as trees of actions, on which the trace of the interaction is easily seen. We might also use the proof-like presentation, that we will focus in the following section.

4.3 Interpreting rules

The three rules used in the proof-like presentation of designs may be interpreted in dialogical terms. Besides the *daimon* already characterized as the action of stopping the exchange concerning a particular QUD, the positive and negative rules may be interpreted as respectively the actions of *making an intervention* and *expecting reactions from the addressee*, respectively. By conforming to the positive rule, the speaker selects a locus among those available from the current history of the dialogue, and s/he decides in what way the content of his/her intervention offers new loci for the addressee.

Example 4 Suppose that during a conversation, it is A's turn of speech, and that he decides to talk about his next holidays by saying U: "*This year, for my holidays, I will go to the Alps, with friends, and by walking*".

[11] We omit ramifications because they may be deduced from the available information.

Let us assume that only the three final ingredients ("to the Alps, with friends, and by walking") are presented as questions under discussion (which is really the case if the other speaker puts the focus only on them, something which can be observed afterwards). This intervention realizes the use of the following positive rule[12]:

$$\dfrac{\begin{array}{ccc}\vdots & \vdots & \vdots \\ 0.1 \vdash \Delta_1 & 0.2 \vdash \Delta_2 & 0.3 \vdash \Delta_3\end{array}}{\vdash 0, \Delta}\,(+0,\{1,2,3\})$$

$\Delta \cup \{0\}$ represents the set of available subjects of conversation, or *questions under discussion* in Ginzburg's terminology, and 0 a locus to anchor the utterance U. Uttering U gives to the addressee the possibility of continuing the exchange by talking about the speaker's destination, or by talking about her travel companions, or about her way of travelling. This is represented by the creation of three subloci[13] of the initial one: 0.1, 0.2 and 0.3. Finally, the intervention *is* the positive action $(+, 0, \{1, 2, 3\})$.

By conforming to the negative rule, in a receptive attitude, the speaker who just made an intervention, has to be ready to receive an intervention of her/his addressee based on one of the loci that her/his intervention opened.

Example 5 In the previous example, the speaker plans, for each locus 0.1, 0.2, 0.3 various ways, for the addressee, to continue the dialogue.

$$\dfrac{\dfrac{\vdash 0.1.I_1^1, \Delta_1 \ldots \vdash 0.1.I_1^{n_1}, \Delta_1}{0.1 \vdash \Delta_1}(-,0.1,\mathcal{N}_1) \quad \dfrac{\vdash 0.2.I_2^1, \Delta_2 \ldots \vdash 0.2.I_2^{n_2}, \Delta_2}{0.2 \vdash \Delta_2}(-,0.2,\mathcal{N}_2) \quad \dfrac{\vdash 0.3.I_3, \Delta_3 \ldots \vdash 0.3.I_3^{n_3}, \Delta_3}{0.3 \vdash \Delta_3}(-,0.3,\mathcal{N}_3)}{\vdash 0, \Delta}(+,0,1,2,3\})$$

$$\text{where for } i \in \{1, 2, 3\}, \mathcal{N}_i = \{I_i^1, \ldots, I_i^{i_1}\}$$

This action amounts to starting several branches each in itself representing a potential treatment of the *QUD* introduced by the sublocus.

Example 6 Suppose that B prepares herself to respond to an intervention coming from A, on the topic of his holidays.

This "internal" action may be represented by:

$$\dfrac{\begin{array}{ccccc}\vdots & & \vdots & & \vdots \\ \vdash 0.I_1 & \ldots & \vdash 0.I_k & \ldots & \vdash 0.I_n\end{array}}{0 \vdash}\,(-,0,\{I_1,\ldots,I_n\})$$

where it is assumed that our set of subloci $\{1, 2, 3\}$ is one of the I_k.

Let us assume that B replies to A: the process initiated by this first negative step is then continued. That means that the dual of the action corresponding to A's first intervention belongs to B's design (otherwise the process would diverge). Moreover, we

12 Where the vertical dots are simply here to underline the expectation of a continuation of the dialogue on these three topics, thus anticipating that the dialogue will be continued only on them.

13 Notice that it is not the unique way to grasp the utterance, it is just a choice among several possible decompositions.

must assume that B's intervention, represented by the action $(+, 0.1, I)$ was planned by A: the action $(-, 0.1, I)$ was indeed planned above the locus 0.1 in A's design. This gives an interaction between the two following designs (in proof-like presentation):

$$\dfrac{\dfrac{\vdash 0.1.1, \ldots, 0.1.n, \Delta_1 \ldots \vdash 01.I_1^{n1}, \Delta_1}{0.1 \vdash \Delta_1}(-,0,\{\mathcal{N}_1,..,\mathcal{N}_n\}) \quad \begin{matrix}\vdots \\ 0.2 \vdash \Delta_2\end{matrix} \quad \begin{matrix}\vdots \\ 0.3 \vdash \Delta_3\end{matrix}}{\vdash 0, \Delta}(+,0,\{1,2,3\})$$

$$\dfrac{\dfrac{\begin{matrix}\vdots \\ 0.1.1 \vdash\end{matrix} \ldots \begin{matrix}\vdots \\ 0.1.n \vdash\end{matrix}}{\vdash 0.1, 0.2, 0.3}(+,0.1,\{1,..,n\})}{0 \vdash}(-,0,\{1,2,3\})$$

This situation could be realized by the following dialogue:

– *This year, for my holidays, I will go* <u>*to the Alps*</u>$_{0.1}$ <u>*with friends*</u>$_{0.2}$ *and* <u>*by walking*</u>$_{0.3}$*,*

– *well, in the Alps, there are a lot of winter entertainments,* <u>*skiing*</u>$_{0.1.1}$*,* <u>*cross-country skiing*</u>$_{0.1.2}$*,* <u>*bobsleigh*</u>$_{0.1.3}$ *....*

After the intervention of the first speaker, the second one selected the first locus introduced (0.1: "*in the Alps*") and decided to elaborate on it by introducing subloci corresponding to various sub-QUDs (0.1.1, 0.1.2, 0.1.2, ...), while the first speaker, on his side, had planned such topics ($\vdash 0.1.1, \ldots, 0.1.n, \Delta_1$). This is of course a strong assumption. Did A not planned all the future topics, that would result in changing the designs according to some definite procedure, *see below*.

4.4 Dialogues as confrontations of frames

The dialogue may continue in various ways. One way is the acceptance by one of the two participants that the "game" is over (for instance she acknowledges that she got enough information, or she accepts the argument of the other participant), this is branded by the positive rule †. If the dialogue can go on until such an end, it is because all the conditions for the convergence of normalization are met. In such a case, *the normalization process itself is seen as the execution of the dialogue*. An alternative issue is provided by the case where a positive action introduces an answer or a QUD which is not included into the expectations of the other participant. In this case, the normalization process fails. Nevertheless, such a breakdown can be repaired in a dynamic way, by (literally) *changing the rules* used by the players (for instance by extending the span of the expected answers or QUD's). This issue will be examined in the next section.

In fact, this alternative issue reveals a particular way of conceiving dialogue: as if each player already has in mind the spectrum of all possible scenarios. This conception meets the position expressed by Changeux (2004):

> Human communication generally takes place in a well defined context of knowledge in which speakers are informing each other [...] Aiming at maximizing the efficiency of communication, each speaker tries to recognize and to infer the intention of the one who communicates. In other words, when communication begins, each partner has in his or her own mind the whole possible content of the speech, which constitutes a subset of all his or her knowledge on the world. [...] We may think that each speaker constantly tries to project his or her frame of thought into the mind of his or her co-speaker.

We shall therefore call the part of any dialogue which consists of the design of one speaker a *dialogue frame*. This leads to the following definition:

Definition 8 (*Dialogue Frames*) A *dialogue frame* is defined as a design where:

- the base contains all the loci on which the speaker may anchor an intervention in positive positions, that is a list of initial QUD's.
- the negative position is either empty (if the speaker initiates the dialogue) or contains the locus where the addressee anchors her first intervention.

The dialogue itself is the trace of the interaction between the two dialogue frames, as seen in Definition 5.

4.5 Dialogues in progress

From the perspective of an external observer, dialogue frames are not generally known. Therefore, they must be retrieved through the observation of a dialogue in progress. In this case, each speaker reveals her strategy step by step, intervention by intervention, action by action. Moreover she adapts her strategy according to the responses of her addressee.

Ginzburg et al. (2009) considers conversations as a "collection of dynamically changing, coupled information states, one per conversational participant." In Ginzburg's terms, the *Dialogue Gameboard* (**DGB**) represents information that arises from publicized interactions. On a similar line of thought, we distinguish *three intertwined streams* (see Fig. 3[14]): the stream of the first speaker's interventions (represented by a design either directly extracted from her/his strategy or adapted from it), the stream of the second interlocutor's interventions, and *the stream of the current states of the interaction*. This latter stream represents the succession of information states resulting from the present interaction and may be associated with the notion of *Dialogue Gameboard*. The first intervention provides the first state ; the normalization between each current state and the next intervention gives the next current state, and so on ...The resulting dialogue is then the trace of the interaction between the two interacting designs which emerge.

[14] Where S is the speaker and A the addressee.

Fig. 3 Dialogue in process

Intervention of *S*	**Current state**	**Intervention of** *A*
$\mathfrak{S}_1$		
	$\mathfrak{E}_1 = \mathfrak{S}_1$	
		$\mathfrak{A}_2$
	$\mathfrak{E}_2 = [[\mathfrak{E}_1, \mathfrak{A}_2]]$	
$\mathfrak{S}_3$		
	$\mathfrak{E}_3 = [[\mathfrak{E}_2, \mathfrak{S}_3]]$	
⋮	⋮	⋮

5 Dialogical relations

We are thus led to a conception in which processes and among them, discursive processes may be seen as two-sided processes: they are determined simultaneously by the decisions the speaker takes concerning her speech and by the expectations of the other speaker, as they are planned by the speaker herself. If it happens that the reactions of the other speaker do not correspond to the expectations of the first speaker, the result is a divergence. If we assume speakers wish to maintain the interaction as long as possible, they are obliged to adapt to each other, trying to plan the reactions of the other speaker.

We argue, then, that discursive relations such as *topicalization*, *elaboration*, *narration* (as described by Asher and Lascarides (2003)), as well as speech acts such as *assertion*, *question* and *denegation* may result from the complex relationships between (inter)locutors which appear in dialogue games. Some of these interlocutors may be virtual: this is the case in monological discourses where the speaker has always in mind a *potential image of her addressee*.

5.1 Topicalization

In most theories of discourse, discourse is analyzed as a hierarchical structure between constituents which are related either by *dominance* or by *precedence* relations (Grosz and Sidner 1986). A *discourse topic* is a root of such a structure. We therefore define the operation of *topicalization* as an operation which aims at providing a common root for several segments of discourse.[15] Asher and Lascarides (2003) gives the following example:

a. John had a great evening last night.
b. He had a great meal.

[15] This notion of "topicalization" refers to the global discourse and is about signalling discourse topic. It must be distinguished from "topicalization" as used in the linguistic litterature, which refers to the placement of constituents sentence-initially.

c. *He ate salmon.*
d. *He devoured lot of cheese.*
e. *He then won a dancing competition.*

In this example, the discourse topic is provided by (a): *John had a great evening last night.*

This is of course not a *dialogue*. Nevertheless, we assume a monological discourse is always coping with a *counter discourse* which could be held either by the speaker himself or by a real interlocutor.

Let us consider designs developing from positive forks with an arbitrary *finite* number of loci: $\vdash \Lambda$. The first action in this case is necessarily positive. It consists in choosing a *focus*. Let ξ be this focus, then the basis may be written as $\vdash \xi, \Lambda_0$. Λ_0 is said to be the *context* in which the QUD ξ is developed. Λ_0 is the reservoir of topics to be addressed during the dialogue. It may be underspecified at the beginning of the interaction. A design based on $\vdash \xi, \Lambda_0$ may always be seen as a subdesign of some design based on a single locus: $\vdash \tau$ provided that the loci ξ, Λ_0 may be rewritten as $\xi, \Lambda_0 = \tau.0.0, \tau.0.1, \ldots.\tau.0.n$. We simply add new steps before the current one in order to make explicit the successive actions which create the loci $\tau.0, \ldots, \tau_n$ starting from a single locus τ. Going downward then unifies the *focus* and its *context* in a single *discourse topic*. This can be seen as a primitive operation of discourse, which deserves to be named *topicalization*. The operation therefore consists of:

$$\dfrac{\dfrac{\vdash \tau.0.0., \ldots \tau.0.n = \Lambda}{\tau.0 \vdash}\,(-, \tau.0, \{I\})}{\vdash \tau}\,(+, \tau, \{0\})$$

where the first step is considered virtual (as if the constitution of the topic came from an expectation from the other, potentially fictitious, speaker). The first player, that is A in this example, thus **changes his initial dialogue frame into this one**, in order to be able to continue the exchange with his addressee. In a real dialogue, any intervention by the addressee may change the topic configuration.

Example 7 Suppose a speaker A decides to describe his holidays, saying: “this year, for my holiday, I will go to the Alps, with friends, by walking …” and that his addressee B asks him: “when, actually, will you go on holiday ?”. To prevent a divergence, A has just to enlarge the interaction.

1. Initially the situation is as follows (like in example 4): A decided to describe his holiday, saying “this year, for my holiday, I will go to the Alps, with friends and by walking”, implicitly inducing that B’s comments and questions will be about these topics.

$$\dfrac{01 \vdash \quad 0.2 \vdash \quad 0.3 \vdash}{\vdash 0}\,(+,0,\{1,2,3\})$$
$$\text{A}$$

2. Instead of anchoring her intervention on $\xi.1$, asking A more questions on the description of his holidays, the addressee (B) asks A : “when will you go on holiday ?”. Usually, the date of the holidays is part of the description of them, and

in a standard lexical semantics approach, that would appear here. However, in a discourse, we may figure out that this topic is accidentally ignored, thus compelling the speaker to add it afterwards. A has then to manage a locus from which he may answer to B. A has not only to put a locus ξ from which he can describe his holidays, but also another one, ρ from which he can give the date of these holidays. More precisely A has to replace the fork $\vdash \xi$ by the fork $\vdash \xi, \rho$ and then to unify the loci in the same context: "the topics of A's holidays", simply by setting:

$$\vdash \xi, \rho = \vdash \tau.0.0, \tau.0.1.$$

Then the following design is used. It may be understood as two successive virtual utterances "I can talk about my holidays" (corresponding to the action $(+, \tau, \{0\})$ and "I am then ready to answer questions about its date, its description" (corresponding to the action $(-, \tau.0, \{0, 1\})$).

$$\dfrac{\dfrac{\vdash \tau.0.0,\ \tau.0.1}{\tau.0 \vdash}\,{-,\tau.0,\{0,1\})}}{\vdash \tau}\,(+,\tau,\{0\})$$

A

3. A can answer to the question on the date (with the action $(+, \tau.0.1, \{6\})$.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\tau.0.0.1 \vdash \quad \tau.0.0.2 \vdash \quad \tau.0.0.3 \vdash}{\vdash \tau.0.0}\,(+,\tau.0.0,\{1,2,3\})}{\tau.0.1.6 \vdash \tau.0.0}\,(-,\tau.0.1.6,\{\emptyset\})}{\vdash \tau.0.0,\ \tau.0.1}\,(+,\tau.0.1,\{6\})}{\tau.0 \vdash}\,(-,\tau.0,\{0,1\})}{\vdash \tau}\,(+,\tau,\{0\}) \qquad \dfrac{\dfrac{\tau.0.0 \vdash \qquad \dfrac{\dfrac{}{\vdash \tau 0.1.i}\,(+,\tau.0.1.6,\emptyset)}{\tau.0.1 \vdash}\,(-,\tau.0.1,\{\{i\},i \in \mathbb{N}\})}{\vdash \tau.0}\,(-,\tau.0,\{\{0,1\}\})}{\tau \vdash}\,(-,\tau,\{0\})$$

A B

The left most design contains, above the rule $(-, \tau.0.1.6, \{\emptyset\})$, a subdesign which, up to a delocalisation ($\tau.0.0$ instead of 0), is the same as the design corresponding to the initial situation when A decided to speak about his holidays. In this design, we have the sequence of rules:

$$(+, \tau, \{0\})(-, \tau.0, \{\{0, 1\}\})(+, \tau.0.1, \{6\})(-, \tau.0.1.6, \{\emptyset\})$$

that we may comment. By means of the two first actions, A manages the loci he needs: the one for answering to the question about his holiday's date: $\tau.0.1$ and the one where his previous intervention is anchored, $\tau.0.0$. Then A gives his holiday's date (arbitrary located on $\tau.0.1.6$) and proceeds to the negative action $(-, \tau.0.1.6, \{\emptyset\})$ by which he indicates that there may be no reply to a merely informative claim.

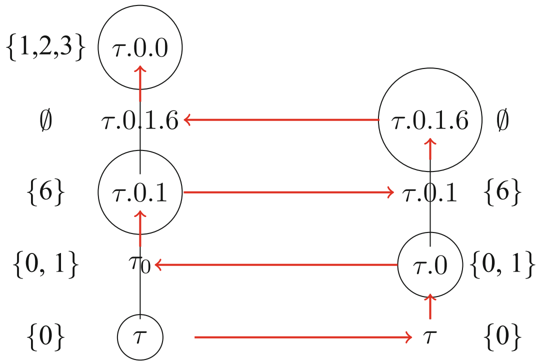
4. And the interaction reduces to:

$$\frac{\tau.0.0.1 \vdash \quad \tau.0.0.2 \vdash \quad \tau.0.0.3 \vdash}{\vdash \tau.0.0}(+, \tau.0, \{1,2,3\})$$

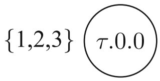
A

where we retrieve the same current state as the one resulting from the first intervention "this year, for my holiday …". A is now waiting for B's questions about the description of his holidays.

Normalization may be represented as follows (using trees of actions):



The travel ends up at:



that is exactly the same situation as the initial one with $\tau.0.0$ instead of ξ.

5.2 Subordinating relations

Asher and Lascarides (2003) intuitively characterize subordinating relations, like *elaboration*, by saying that *they relate two propositions if the second plays a subordinate role relative to the first, for instance elaborating or supporting that idea with arguments.* We can imagine a locus is chosen initially in the positive process (that is, the one the first rule of which is positive) and then each positive step consists of elaborating on that locus or on its subloci without being forced to go to a disjoint locus by the facing negative process (recall that a sublocus of ξ where ξ is an address, that is a sequence of *biases* (integers) is simply a locus a prefix of which is ξ).[16] In this case, we say that the discursive relation between all the utterances is a *subordinating* one.

Example 8 – I go to the mountain.(a_1)
– because I like skiing.(a_2)
– and above all cross-country skiing(a_3)

[16] A disjoint locus should be a locus which has no common prefix with the current one. In fact when staying on a single *topic*, they minimally have the locus of this topic as a common prefix, but we may assume it is the only common prefix they have.

– it's not dangerous if you are careful
– you still have to plan the avalanches
– but fortunately I have an ARVA

The formalization of that discourse, uttered by a locutor A, is provided by the design on the left hand side of the figure below:

$$
\begin{array}{cl}
\vdots & \\
\xi.0.1.0.1.0 \vdash \Delta & \\
\hline
\vdash \xi.0.1.0.1, \Delta & (+,\xi.0.1.0.1,\{0\}) \\
\hline
\xi.0.1.0 \vdash \Delta & (-,\xi.0.1.0,\{\{1\}\}) \\
\hline
\vdash \xi.0.1, \Delta & (+,\xi.0.1,\{0\}) \\
\hline
\xi.0 \vdash \Delta & (-,\xi.0,\{\{1\}\}) \\
\hline
\vdash \xi, \Delta & (+,\xi.0,\{0\})
\end{array}
\qquad\qquad
\begin{array}{cl}
\vdots & \\
\vdash \xi.0.1.0.1.0 & \\
\hline
\xi.0.1.0.1 \vdash & (-,\xi.0.1.0.1,\{\{0\}\}) \\
\hline
\vdash \xi.0.1.0 & (+,\xi.0.1.0,\{1\}) \\
\hline
\xi.0.1 \vdash & (-,\xi.0.1,\{\{0\}\}) \\
\hline
\vdash \xi.0 & (+,\xi.0,\{1\}) \\
\hline
\xi \vdash & (-,\xi,\{\{0\}\})
\end{array}
$$

$$(a_1 = (+, \xi, \{0\}), \quad a_2 = (+, \xi.0.1, \{0\}), \quad a_3 = (+, \xi.0.1.0.1, \{0\}))$$

The right hand side provides an orthogonal process, which is virtual in this case. The important point to notice is that the virtual process with which the speaker is confronted when producing her discourse is as important as her own process. It is because she has this process in mind that she continues her discourse this way. The virtual co-process can be made explicit by a series of questions such as "Why to go to the mountains ?", "Which style of skiing are you practicing ?", "Is it not dangerous ?", which are all questions anchored in loci introduced by the previous statement.

5.3 Coordinating relations

Narration is a precedence relation. It relates two propositions only if the event described by the first proposition temporally precedes that of the second. More generally, coordinating relations relate propositions at the same level of detail. For the time being, we shall not be concerned with temporality in the usual sense, but only with temporality in discourse, and we will so address coordinating relations. We assume that in a coordinating relation, segments are associated with disjoint *loci*, and addressed sequentially, one after the other.

Giving up a QUD translates into the selection of a new locus in the so-called *context*. When all the loci inside the initial context are explored, the result is compatible with a *coordinating relation*.[17]

Example 9
– I went to the mountain during a fortnight (a_1)
– then I took the plane to Frisco (a_2)

[17] We do not argue that a coordinating relation only consists of this formal, geometrical aspect. Additional features are, of course, needed to characterize a text as sequence of propositions related by a coordinating relation, all pertaining to the content.

– from there I visited California
– and then I went back to Europe

The formalization is given by:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dots}{\xi.2.0 \vdash \xi.3, \dots, \xi.n}(-, \xi.2.0, \{\emptyset\})}{\vdash \xi.2, \dots, \xi.n}(+, \xi.2, \{0\})}{\xi.1.0 \vdash \xi.2, \dots, \xi.n}(-, \xi.1.0, \{\emptyset\})}{\vdash \xi.1, \xi.2, \dots, \xi.n}(+, \xi.1, \{0\}) \qquad \dfrac{\dfrac{}{\vdash \xi.1.0}(+, \xi.1.0, \emptyset)}{\xi.1 \vdash}(-, \xi.1, \{0\}) \quad \dots \quad \dfrac{\dfrac{}{\vdash \xi.i.0}(+, \xi.i.0, \emptyset)}{\xi.i \vdash}(-, \xi.i, \{0\}) \quad \dots$$

The dual (family of) design(s) is given on the right, where the only positive steps are labelled by the rule Ø. Again, the other (virtual) speaker has a fundamental role: he instructs the first speaker to abandon a QUD, and to select another one until the range of potential QUD's is exhausted. It may seem that we address only coherent discourses but we may in fact characterize coherence versus incoherence by means of our central concept of convergence. A coherent discourse is convergent with its counter discourse, while an incoherent one is not.[18]

5.4 Assertion and interrogation

The previous rebrands concerning the necessity of a two-sided process for a formalization of discursive relations opens the field to a deeper consideration on elementary speech acts. Discourse is above all action and commitment: action of *Myself* on *Yourself* and reciprocally (Beyssade and Marandin 2006). For instance, as proposed by Walton (2000), *asserting* is "willing to defend the proposition that makes up the content of the assertion, if challenged to do so". This results in the fact that when I assert P, I must have in mind all justifications for predictable objections. That is, "my" design is as follows:

$$\dfrac{\dfrac{\dfrac{\mathfrak{D}_1}{\vdash \xi.0.I_1} \quad \dots \quad \dfrac{\mathfrak{D}_n}{\vdash \xi.0.I_n}}{\xi.0 \vdash}(-, \xi.0, \{I_1, ., I_n\})}{\vdash \xi}(+, \xi, \{0\})$$

where $\{I_1, \dots, I_n\}$ is a set of predictable arguments which can be used to object to the thesis I introduce. *Interrogation* is another game. If other speech acts can always be represented as anchored at a single locus (modulo some "shift" which makes us go downward, in search of the topic or the basis for a denegation), we assume that questions always begin from *two loci*, one of which is called the *locus* of the answer.

[18] More may be elaborated on this question. We may assume that the counter discourse is a kind of *common ground*, and that incoherence means a failure to meet this common ground.

Therefore, the design of a question has a basis $\vdash \xi, \sigma$ with σ devoted to an answer, and ends with $\mathfrak{F}ax_{\sigma}$, so that, in interaction with a dual design $\mathfrak{E}$, the answer to the question is moved to σ.

Example 10 Let us take as examples two elementary dialogues consisting of sequences of *Questions-Answers*, where one is well-formed and the other ill-formed.

The first one is:

– *A: Do you have a car?*

– *B: Yes,*

– *A: What brand is it ?*

It is represented by the following figure:

$$\dfrac{\dfrac{\dfrac{}{\vdash \sigma}\dagger \qquad \dfrac{\dfrac{Fax_{\xi.0.1.0,\sigma}}{\xi.0.1.0 \vdash \sigma}}{\vdash \xi.0.1, \sigma}(+,\xi.0.1,\{0\})}{\xi.0 \vdash \sigma}(-,\xi.0,\{\emptyset,\{1\}\})}{\vdash \xi, \sigma}(+,\xi,\{0\})$$

where the answer “no” is represented by $\emptyset$ (since there is no more to say about the car), the answer “yes” by $\{1\}$, thus creating a locus from which the speaker may continue the interaction on the topic of the car and may, for example, ask about its brand. In such a case the answer will be recorded as a design anchored on the locus σ. This is represented by the following figure where *a* is the answer *“It is a Honda”*:

$$\dfrac{\dfrac{\dfrac{}{\vdash \sigma}\dagger \qquad \dfrac{\dfrac{Fax_{\xi.0.1.0,\sigma}}{\xi.0.1.0 \vdash \sigma}}{\vdash \xi.0.1, \sigma}(+,\xi.0.1,\{0\})}{\xi.0 \vdash \sigma}(-,\xi.0,\{\emptyset,\{1\}\})}{\vdash \xi, \sigma}(+,\xi,\{0\}) \quad \text{vs.} \quad \dfrac{\dfrac{\dfrac{\dfrac{\xi.0.1.0.6 \vdash}{\vdash \xi.0.1.0}(+,\xi.0.1.0,\{6\})}{\xi.0.1 \vdash}(-,\xi.0.1,\{\{0\}\})}{\vdash \xi.0}(+,\xi.0,\{1\})}{\xi \vdash}(-,\xi,\{\{0\}\})$$

The interaction produces:

$$\dfrac{\sigma.6 \vdash}{\vdash \sigma}(+,\sigma,\{6\})$$

which precisely contains the information “B’s car is a Honda”.

The second dialogue is:

– *Do you have a car?*
– *No, I have no car*
– ** What brand is it?*

and it may be represented either on the following figure:

$$\dfrac{\dfrac{\dfrac{\dfrac{Fax_{\xi.0.1.0,\sigma}}{\xi.0.1.0 \vdash \xi.0.1, \sigma}}{\vdash \xi.0.1, \sigma}(+,,\xi.0.1,\{0\})}{\xi.0 \vdash \sigma}(-,\xi.0,\{\{1\}\})}{\vdash \xi, \sigma}(+,\xi\{0\}) \qquad \text{vs.} \qquad \dfrac{\dfrac{}{\vdash \xi.0}(+,\xi.0,\emptyset)}{\xi \vdash}(-,\xi,\{\{0\}\})$$

where the dialogue fails since A did not plan a negative answer, or on the next figure, where the dialogue also fails since A can only play on the left branch, thus confusing the locus σ (which is a place for recording the answer) and the locus $\xi.0$ which corresponds to the fact that the answer would have been “yes”.

$$\dfrac{\dfrac{\dfrac{\xi.0.1.0 \vdash}{\vdash \sigma}(+,\xi.0.1,\{0\}) \quad \dfrac{\dfrac{Fax_{\xi.0.1.0,\sigma}}{\xi.0.1.0 \vdash \sigma}}{\vdash \xi.0.1, \sigma}(+,\xi.0.1,\{0\})}{\xi.0 \vdash \sigma}(-,\xi.0,\{\emptyset,\{1\}\})}{\vdash \xi, \sigma}(+,\xi,\{0\} \qquad \text{vs.} \qquad \dfrac{\dfrac{}{\vdash \xi.0}(+,\xi.0,\emptyset)}{\xi \vdash}(,\xi,\{\{0\}\})$$

6 Conclusion

Ludics provides a frame in which we can explore speech acts realized in discourse as truly *two-sided*. This is mainly because it is based on interaction between parallel processes, generalizing the well-known dynamics of proofs (already present in Gentzen’s sequent calculus, through the *cut-elimination* procedure) to the *dynamics of paraproofs*. In such a framework, there is no *truth* strictly speaking but only ways for a proof-candidate to pass *tests* which are themselves other proof-candidates. In a concrete dialogue situation, our utterance is a proof-candidate: it necessarily has to cope with counter proof-candidates, which are either the reactions of the other speaker or some kind of virtual reaction that we have in mind. In this way, our contributions are doubly-driven: once by our positive acts, and secondly through the positive actions of the other speaker or some virtual partner and through the way we record these reactions, that is through negative acts. Of course, while in a dialogue each participant has to take into consideration the expectations and reactions of the other, in monologues, utterances are co-determined by the speaker herself, and by her virtual interlocutor. It is this interaction which drives the speech until a tacit agreement occurs either coming directly from the speaker or indirectly via the image she has of her speech.

We also assume that concrete dialogues are always concrete manifestations of potential ones. That is, we consider our view to be coherent with a position that takes for granted that any utterance commits its speaker to provide reasons and justifications for what she is saying. This entails a conception of semantics which is different from the usual *referential* framework and seems to involve an *inferentialist* semantics of the kind argued for by Brandom (2000). This interaction between inferentialist semantics and dialogue pragmatics will be the topic of our future works.

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