Dialogue and Interaction : the Ludics view

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Abstract

In this paper, we study dialogue as a game, but not in the sense in which there would exist winning strategies and a priori rules. On the contrary, it is possible to play with rules, provided that some geometrical constraints are respected (orthogonality). We owe much to Ludics, a frame developed by J-Y Girard, while remaining close to the approach of discourse and dialogue in N. Asher's tradition. We interpret the ludical notion of *locus* in rhetoric terms, as a location in a discourse from which a particular theme is developed. A set of *loci* subordinated to a same initial locus is a *topic*. We explain the processes of *narration* and *elaboration*, and also the speech acts of *assertion, denegation* and *interrogation* in ludical terms.

1 Introduction

In what follows, we shall refer to *Ludics*, a theory elaborated by J-Y. Girard (Girard 01, 03, 06) in the goal of reconstructing logic starting from the notion of *interaction*. We think that this frame is particularly suitable for representing *dialogues*. In a word, it starts from the observation that proofs in a polarized logic (as Linear Logic may be seen) can be presented as processes which make alternate negative and positive steps. This observation opens the field to the concept of duality between abstract processes generally called *designs*. These objects have in fact two readings: one is as proofs, and the other as strategies in a game. We may think that the *proof* aspect is convenient for dialogues in that it represents the *argumentative* content of a statement. The *strategy* aspect is convenient also in that it involves goals and directions towards which a dialogue is oriented. With regards to other game theories, in Ludics, rules are not a priori given, interaction itself determines them. Otherwise, each step in a play records all the previous ones, thus allowing more flexibility, in particular for backtracking during a discussion.
The semantic which can be given, via *Ludics* to utterances is not simply truth-conditional. We may start from our intuitive notion of what it is for a piece of dialogue (or of discourse) to be "well formed", to give rise to elementary situations of interaction, thus suggesting another (empirical) view on Semantics [Lecomte-Quatrini10]. In this presentation we shall make a link between Ludics and the well known notions of SDRT (Asher & Lascarides) applied to dialogue as an example of the expressivity of Ludics.

2 Interaction as a basis

$$\dfrac{\begin{matrix}\vdots & & \vdots\\ 0 \star 1_k \vdash \Delta_{1_k} & \ldots & 0 \star n_k \vdash \Delta_{n_k}\end{matrix}}{\vdash 0, \Delta} \qquad \dfrac{\begin{matrix}\vdots & & \vdots & & \vdots\\ \vdash 0 \star I_1, \Gamma & \ldots & \vdash 0 \star I_k, \Gamma & \ldots & \vdash 0 \star I_n, \Gamma\end{matrix}}{0 \vdash \Gamma}$$

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$$\dfrac{\begin{array}{c}\vdots\\ \vdash 0\star i_k\star 1,\ldots,0\star i_k\star n,\Delta_{i_k}\end{array}}{0\star i_k\vdash\Delta_{i_k}} \qquad \dfrac{\begin{array}{ccc}\vdots & & \vdots\\ 0\star i_k\star 1\vdash\Gamma_1 & \ldots & 0\star i_k\star n\vdash\Gamma_n\end{array}}{\vdash 0\star I_k,\Gamma}$$

Figure 1: Continuation right and left

The archetypal figure of interaction is provided by two intertwined processes the successive times of which, alternatively positive and negative, are opposed by pairs. On the left, we have a process starting from a set of *loci*[1] that we assume to be "positive", among which there is one which is focused, here denoted by '0', and at the first step, this locus is made to vary across the various manners a given theme may be addressed. Each of these issues selects a subset of *loci* among the remaining (not focused) ones, thus showing that, according to the way the theme is addressed, various subthemes may be discussed later.
On the right, we have a second process, for which a locus has been already chosen, and therefore put in a negative position (the left hand side of the so-called "fork", representing in fact a sequent with at most one locus on the left hand side). This represents a receptive attitude : the locus is the one which has been selected in the other process. The first step of this process consists in a survey of all the various ways it could be possible to decline this locus (to address this theme). Among them, if things are going well, there is the one taken in the first process. Such a configuration may be associated with *cooperation*: both processes have a dialogue together, and we may imagine it lasts some time. Let us admit for instance that the right process be continued, it records the positive action of the left one and it is now its turn to perform a positive action, but the left process must have planned this action. This gives the figure 1[2]. This situation could be illustrated by the following dialogue :
- *This year, for my holidays, I will go to the Alps with friends and by walking,*
- *well, in the Alps, there are a lot of winter entertainments*
The first speaker extracts from a set of *loci*, a *locus* '0' associated with the topic of her holidays, that she may topicalize by asserting something on *where* she goes, *with whom*, *by what means* and so on. The second speaker records the theme of holidays and is ready to accept various ways of addressing it (a set of sets of *loci* that we shall call a set of *thematic variations*). After accepting the way her interlocutor addresses it, she may focalize on one of the aspects the first speaker introduced, for instance here on the *where*, thus introducing various ways of topicalizing it. The first speaker has already in mind a whole directory of possible thematic variations concerning the point developed by the second one.
In Ludics, such an interaction may continue in various ways. One way is the acceptance by one of the two participants that the "game" is over (for instance she acknowledges that she got enough information, or she accepts the argument of the other participant), this is marked by the positive rule †. If such a case occurs and if all the *loci* introduced in the dialogue have been visited by the normalization process[3] (in this case, the set of both processes is said to form a *closed net*), the two processes are said to be *orthogonal* or that they *converge*. An alternative issue is provided by the case where a positive action introduces a thematic variation which is not included into the expectations of the other participant: the interaction is said to *diverge*. We shall see in a future paper that such a breakdown may be repaired in a dynamic way, by (litterally) *changing the rules* the players are using (for instance by extending the span of the expected thematic variations).

[1]a *locus* is a mere address like a memory cell or like a specific position occupied by a statement in a conversational network.
[2]$i\star J$, where J is a set of indexes $\{j_1, j_2, ..., j_m\}$ means $\{i\star j_1, i\star j_2, ..., i\star j_m\}$.
[3]That is, at each step of the confrontation, a positive locus is put in correspondance with a negative one, and all the negative loci are finally "cancelled" by their positive counterparts

3 Dialogical relations

3.1 Topicalization

We will mainly consider designs developing from positive forks with an arbitrary finite number of loci : $\vdash \Lambda$. The first action in this case is necessarily positive and consists in choosing a *focus*. Let ξ be this focus, then the basis may be written $\vdash \xi, \Lambda_0$. Λ_0 is said to be the *context* in which the theme ξ is developed. Of course, we may have always a single locus, by exploring the design in the top-down direction (instead of the bottom-up one). That involves to introduce *pre-steps* to the current one. This is always possible provided that the loci ξ, Λ_0 may be rewritten as $\xi = \tau \star 0 \star 0, \tau \star 0 \star 1, \cdots \star \tau \star 0 \star n$. Going downward like this unifies the focus and its context in a *topic*. This can be seen as a primitive operation of discourse, which deserves to be named *topicalization*. The operation consists in:

$$\cfrac{\cfrac{\vdash \tau \star 0 \star 0., \ldots \tau \star 0 \star n = \Lambda}{\tau \star 0 \vdash}\,(-, \tau \star 0, \{I\})}{\vdash \tau}\,(<>, +, \{0\})$$

where we consider the first step as virtual (as if the constitution of the topic came from an expectation from the Other Speaker).

EXAMPLE

Suppose that you decide to describe your next holiday : “this year, for my holiday, I will go to the Alps, with friends, by walking ...” and that your addressee asks you: “when are you on holiday ?”. If you do not wish to abandon at once, that is, if you wish to go on interacting with him/her, you have just to enlarge the interaction.

1. First step you decide to describe your next holiday : “this year, for my holiday, I will go to the Alps ...”

$$\frac{\xi \star 1 \vdash}{\vdash \xi}$$

(Y)our intervention

2. Instead of anchoring her intervention on $\xi \star 1$, asking you more questions on the description of your holiday, your addressee asks you: “when are you on holiday ?”
Then, you have to manage a locus from which you may answer to him/her *in the same interaction* as the one you started with ; you have not only to put a locus ξ from which you can tell the description of your holidays but also another one, ρ, from which you can tell the date of your holidays. More precisely you have to replace the fork $\vdash \xi$ by the fork $\vdash \xi, \rho$ and then to unify the loci in the same context: “the topics of your holiday”, simply by setting:

$$\vdash \xi, \rho \quad = \quad \vdash \tau \star 0 \star 0, \tau \star 0 \star 1.$$

Then, the following design is used. It may be understood as two successive virtual utterances “I can tell you something about my holiday” (corresponding to the action $(+, \tau, \{0\})$ and “I am then ready to answer any questions about the dates and its description” (corresponding to the action $(-, \tau \star 0, \{0, 1\})$).

$$\cfrac{\cfrac{\vdash \tau \star 0 \star 0, \tau \star 0 \star 1}{\tau.0 \vdash}}{\vdash \tau}$$

Y

3. you can then answer to the question on the dates (with the action $(+, \tau \star 0 \star 1, \{6\})$.

$$\begin{array}{c} \tau\star 0\star 0\star 1\vdash \\ \hline \vdash \tau\star 0\star 0 \\ \\ \hline {}^{\emptyset} \\ \tau\star 0\star 1\star 6\vdash \tau\star 0\star 0 \\ \hline \vdash \tau\star 0\star 0, \tau\star 0\star 1 \\ \hline \tau.0\vdash \\ \hline \vdash\tau \\ \text{Y} \end{array} \qquad \begin{array}{c} \dfrac{\tau\star 0\star 0\vdash \qquad \dfrac{\dfrac{}{\vdash \tau 0\star 0\star 1\star i}\,\emptyset}{\tau\star 0\star 1\vdash}\,{\scriptstyle i\in\mathbb{N}}}{\dfrac{\vdash\tau\star 0}{\tau\vdash}} \\ \text{A} \end{array}$$

4. The foregoing interaction reduces to:

$$\begin{array}{c} \dfrac{\tau\star 0\star 0\star 1\vdash}{\vdash\tau\star 0\star 0} \\ \text{Y} \end{array} \qquad \begin{array}{c} \dfrac{\vdash\tau\star 0\star 0\star 1}{\tau\star 0\star 0\vdash} \\ \text{A} \end{array}$$

The situation is hence the same as the one which resulted from your first intervention "this year, for my holiday …". You are now waiting for your addressee's questions about the description of your holiday.

3.2 Elaboration

We may imagine a locus be chosen initially in the positive process (that is the one the first rule of which is positive) and then each positive step consists in elaborating on that locus or on its sub-loci (a sub-locus of ξ where ξ is an address, that is a sequence of *biases* (integers) is simply a locus a prefix of which is ξ), without the facing negative process forcing it to go to a disjoint locus[4]. In this case, we say that the discursive relation between all the utterances made is *elaboration*. An example is provided by:
- *I go to the mountain.(a_1)*
- *I like skiing.(a_2)*
- *above all cross-country skiing*
- *it's not dangerous if you are careful*
- *you have nevertheless to plan the avalanches*
- *but fortunately I have a NARVA*

The formal representation of that discourse is provided by the design on the left hand side of figure 2.The right hand side provides an orthogonal process. This is a virtual process. The important point to notice here is that the virtual process to which the Speaker is confronted when producing her discourse is as important as her own process of speaking. It is because she has in mind this process that she continues her discourse this way. For example she adds some successive virtual questions as "Why going to the mountain ?" ; "Which style of skiing ?" ; "Is it not dangerous ?" …That would be a different thing if the virtual process in one of its positive actions had selected a locus in Δ (for instance at the second step the question could have been : "To the Alsp or to other mountains ?").

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dots}{\xi_{0i0}\vdash\xi_{01},\dots,\xi_{0n},\Delta}}{\vdash\xi_{01},\dots,\xi_{0i},\dots,\xi_{0n},\Delta}\,a_2}{\xi_0\vdash\Delta}}{\vdash\xi,\Delta}\,a_1}{\texttt{TOP} : holidays} \qquad \dfrac{\xi_{01}\vdash\Delta_1,\dots \qquad \dfrac{\dfrac{\dots}{\vdash\xi_{0i0},\dots,\xi_{0im},\Delta_i}}{\xi_{0i}\vdash\Delta_i} \qquad \xi_{0n}\vdash\Delta_n}{\dfrac{\vdash\xi_0,\Delta}{\xi\vdash\Delta}}$$

[4] a disjoint locus should be a locus which has no common prefix with the current one, in fact when staying inside a same *topic*, they have at least the locus of this topic as a common prefix, but we may assume it is the only common prefix they have.

Figure 2: Elaboration

3.3 Narration

When a theme is given up, this translates into a new locus being selected in the so-called *context*. When all the loci inside the initial context are explored, this results in a *narration*. Let us see an example:
- *I went to the mountain (a_1)*
- *then I took the plane to Frisco (a_2)*
- *from there I visited California*
- *and then I went back to Europe*

The formal representation is given by figure 3,

$$\cfrac{\cfrac{\cfrac{\cfrac{...}{\xi_{20} \vdash \xi_3, ..., \xi_n}\{\emptyset\}}{\vdash \xi_2, ..., \xi_n}\, a_2}{\xi_{10} \vdash \xi_2, ..., \xi_n}\{\emptyset\}}{\vdash \xi_1, \xi_2, ..., \xi_n}\, a_1 \qquad\qquad \cfrac{\cfrac{}{\vdash \xi_{10}}\,\emptyset}{\xi_1 \vdash} \qquad ... \qquad \cfrac{\cfrac{}{\vdash \xi_{i0}}\,\emptyset}{\xi_i \vdash} \qquad ...$$

Figure 3: Narration

where $\{\emptyset\}$ denotes a particular case of the negative rule. The dual (family of) design(s) is given on the right, where the only positive steps are labelled by the rule $\emptyset$. Again, the other (virtual) speaker has a fundamental role : she determines the first speaker not to develop a theme, and to select another one until the range of themes be exhausted.

3.4 Assertion, denegation and interrogation

The previous remarks concerning the necessity of a two-faces process for a representation of discursive relations opens the field to a deeper reflection on elementary speech acts. Discourse is above all action and commitment : action of *Myself* on *Yourself* and reciprocally [Beyssade & Marandin 06]. Like it is said by Walton [Walton 00], “asserting” is “willing to defend the proposition that makes up the content of the assertion, if challenged to do so” . This results in the fact that when *Myself* asserts P, I must have in mind all justifications for predictable objections. That is, “I” have a design like in figure 4

$$\cfrac{\cfrac{\cfrac{\mathcal{D}_1}{\vdash \xi_0.I_1} \;...\; \cfrac{\mathcal{D}_n}{\vdash \xi_0.I_n}}{\xi_0 \vdash}\,\mathcal{N}}{\vdash \xi}$$

Figure 4 : Assertion

where $\mathcal{N}$ is a set of predictable thematic variations on the theme “I” introduce, and where every $\mathcal{D}_i$ is a design which never ends up by a †.

Denegation is slightly different. We can refer to works by O. Ducrot in the eighties [Ducrot 1984] according to whom discursive negation (that we shall name denegation in the present paper in order to avoid confusion

with standard logical negation) is necessarily *polyphonic*. We may for instance have the following utterance in a dialogue:
- Mary is not nice, on the contrary, she is execrable
where the second part of the utterance may be understood only if we think that it denegates not the first part of the sentence but a dual utterance, the one which the present one is confronted with and which could be *Mary is nice*. We are therefore led to conclude that a (de)negation like *Mary is not nice* is always opposed to a virtual positive statement like *Mary is nice*. A possible modeling of such a case consists in having a positive action by *Myself* which compels the other speaker to accept my denegation by playing her †. If not, she enters into a diverging process[5]. The only way for *Myself* to force the other speaker to play her † is to use the $\emptyset$ positive rule. On the other hand, the denegation assumes that the virtual other speaker produced a statement which is now denied by *Myself*. This statement is in fact a paradoxal assertion since the set $\mathcal{N}$ is reduced to $\{\emptyset\}$! (The virtual speaker has no plan to sustain the claim she makes). Denegation therefore supposes we make a step downward, to the fictitious claim (see figure 5)

$$\dfrac{}{\vdash \xi, \Lambda}\,\emptyset \qquad\qquad vs \qquad\qquad \dfrac{\dfrac{}{\vdash \Gamma}\,\dagger}{\xi \vdash \Gamma}$$

$$\downarrow$$

$$\dfrac{\dfrac{}{\vdash \xi_0, \Lambda}\,\emptyset}{\xi \vdash \Lambda}\,\{\{0\}\} \qquad\qquad vs \qquad\qquad \dfrac{\dfrac{\dfrac{}{\vdash \Gamma}\,\dagger}{\xi_0 \vdash \Gamma}}{\vdash \xi, \Gamma}\,\{0\}$$

Figure 5 : Denegation

Interrogation is still another game. If other speech acts can always be represented as anchored at a single locus (modulo some "shift" which makes us going downward, searching the topic or the basis for a denegation), we assume questions always starting from *two loci*, among which one is called the *locus* of the answer. The design of a question has therefore a basis $\vdash \xi, \sigma$ with σ devoted to an answer, and ends up by a $\mathcal{F}ax_\sigma$, so that, in interaction with a dual design $\mathcal{E}$, the answer to the question is moved to σ. Let us now take as examples two elementary dialogues consisting of sequences of **Questions-Answers**, where one is well formed and the other ill formed.
The first one is : *- YOU : Have you a car?*
- I : Yes,
- YOU : Of what mark?
It is represented on figure 6

$$\dfrac{\dfrac{\dfrac{}{\vdash \sigma}\,\dagger \qquad \dfrac{\dfrac{Fax_{\xi010,\sigma}}{\xi010 \vdash \sigma}}{\vdash \xi01, \sigma}\,{\scriptstyle You_3}}{\xi0 \vdash \sigma}\,{\scriptstyle \{\emptyset,\{1\}\}}}{\vdash \xi, \sigma}\,{\scriptstyle You_1} \qquad\qquad \text{vs} \qquad\qquad \dfrac{\dfrac{\dfrac{\vdots}{\vdash \xi010}}{\xi01 \vdash}}{\dfrac{\vdash \xi0}{\xi \vdash}}$$

[5] that coud be repaired in a dynamic way.

Figure 6 : First dialogue

The answer ”yes” is represented by by $\{1\}$, creating hence a locus from which the speaker may continue the interaction on the car’s topic and for example may ask which is its mark.
The answer ”no” is represented by $\emptyset$ (there is no more to say about this car) .
The second dialogue is :
- Have you a car?
- No, I have no car
- * *Of what mark?*
and it may be represented either on figure 7 where the dialogue fails since YOU did not planified a negative answer, or on figure 8 where the dialogue also fails since YOU can only play on “your” left branch, thus confusing the locus σ (which is a place for recording the answer) and the locus $\xi.0$ which corresponds to the fact that the answer would have been “yes”.

$$\dfrac{\dfrac{\dfrac{\dfrac{Fax_{\xi010,\sigma}}{\xi010 \vdash \xi01, \sigma}}{\vdash \xi01, \sigma}\,{\scriptstyle You_3}}{\xi0 \vdash \sigma}\,{\scriptstyle \{\{1\}\}}}{\vdash \xi, \sigma}\,{\scriptstyle You_1} \qquad \text{vs} \qquad \dfrac{\dfrac{}{\vdash \xi0}\,{\scriptstyle \emptyset}}{\xi \vdash}$$

Figure 7 : Second dialogue-1

$$\dfrac{\dfrac{\dfrac{\xi010 \vdash}{\vdash \sigma} \quad \dfrac{\dfrac{Fax_{\xi010,\sigma}}{\xi010 \vdash \sigma}}{\vdash \xi01, \sigma}\,{\scriptstyle You_3}}{\xi0 \vdash \sigma}\,{\scriptstyle \{\emptyset,\{1\}\}}}{\vdash \xi, \sigma}\,{\scriptstyle You_1} \qquad \text{vs} \qquad \dfrac{\dfrac{}{\vdash \xi0}\,{\scriptstyle \emptyset}}{\xi \vdash}$$

Figure 8 : Second dialogue-2

4 Conclusion

Ludics provides a frame in which we can explore the speech acts realized in discourse as really *two-faces*. This is mainly because Ludics, as a locative framework, makes it possible to make interact two parallel processes, thus generalizing the well known dynamics of proofs (that we have already in Gentzen’s sequent calculus, by means of the procedure of *cut-elimination*) to the *dynamics of paraproofs* (see the Annex 5.3). In such a framework, there is no *truth* properly speaking but only ways for a proof-candidate to pass *tests* which are themselves other proof-candidates. In a concrete dialogue situation, our utterance is a proof-candidate : it has necessarily to cope with counter proof-candidates, which are either the reactions of the other speaker or some kind of virtual reaction that we have in mind. This way, our interventions are doubly driven: once by our positive acts, and secondly by the positive actions of the other speaker or of such a virtual partner and by the way we record these reactions, that is by negative acts. Of course, while in a dialogue

each participant has to take into consideration the expectations and reactions of the other, in monologues, utterances are co-determined by the speaker herself, and by her virtual interlocutor. It is this interaction which drives the speech until a tacit agreement occurs either coming directly from the speaker or indirectly via the image she has of her speech.
We also assume that concrete dialogues are always concrete manifestations of potential ones, that is we think our view is plainly coherent with one which takes for granted that any utterance commits its speaker to give reasons and justifications for saying it. This entail a conception of semantics which is different from the usual *referential* framework and seems to involve an *inferential* semantics of the kind R. Brandom argues for [Brandom 00]. This articulation between inferential semantics and dialogue pragmatics will be the topic of our future works.

5 Annex: a very short presentation of Ludics

Ludics is a recent theory of Logic introduced by J.-Y. Girard in [Girard 01]. We introduce below some of its mains notions.

5.1 Proofs as processes

Let us start from a particular formulation of Linear Logic : *Hypersequentialized Linear Sequent Calculus*. This formulation elaborates on the fact that Linear Logic may be polarized, that is, we have positive connectives ($\otimes$ and $\oplus$), which are also said active, in the sense that they make non reversible choices in the construction of a proof, and negative ones (& and $\wp$) which don't. By grouping together successive positive (resp. negative) steps, it is possible to present a proof as an alternation of positive and negative steps. A logic results, which has only two "logical" rules: one positive and the other negative. It also has a cut rule and axioms.

$$\frac{\vdash A_{11}, \ldots, A_{1n_1}, \Gamma \quad \ldots \quad \vdash A_{p1}, \ldots, A_{pn_p}, \Gamma}{(A_{11}^{\perp} \otimes \cdots \otimes A_{1n_1}^{\perp}) \oplus \cdots \oplus (A_{p1}^{\perp} \otimes \cdots \otimes A_{pn_p}^{\perp}) \vdash \Gamma}$$

$$\frac{A_{i1} \vdash \Gamma_1 \quad \ldots A_{in_i} \vdash \Gamma_p}{\vdash (A_{11}^{\perp} \otimes \cdots \otimes A_{1n_1}^{\perp}) \oplus \cdots \oplus (A_{p1}^{\perp} \otimes \cdots \otimes A_{pn_p}^{\perp}), \Gamma}$$

où $\cup\Gamma_k \subset \Gamma$[6] and, for $k, l \in \{1, \ldots p\}$, $\Gamma_k \cap \Gamma_l = \emptyset$.
The first of these two rules is the negative one. A negative formula (left-hand side of the sequent) all the subformulae of which are combined by positive connectives (if it were on the right hand side, the connectives would be replaced by $\wp$ et &, which are negative) happens to be decomposed in a canonical way when applying this rule: there is no particular choice to make.
The second one is the positive rule. A positive formula all the sub-formulae of which are combined by positive connectives happens to be decomposed according to some possible choices.
We also have, as usual, the cut-rule :

$$\frac{A \vdash B, \Delta \qquad B \vdash \Gamma}{A \vdash \Delta, \Gamma}$$

[6]The fact that $\cup_k \Gamma_k$ can be strictly included into Γ allows to retrieve weakening.

Unary operators called *shifts* allow changes of polarities of formulae, thus permitting to break a big step into several smaller ones. As may be easily seen on another hand, it is possible to present this calculus by using only "forks ", that is sequents of the form $A \vdash \Delta$ with the left hand side possibly empty, where all the formulae are positive (negative ones being transferred on the left after they have made positive).

5.2 Locativity

A remarkable property of Linear Logic resides in the ability it provides to represent a proof by a graph, or net, simply called *proofnet*. Starting from a sequent to demonstrate, we decompose its formulae until we reach atoms and according to their polarity and the types of links by which their two main subformulae are combined, then, positive and negative instances of the same atoms are connected (axiom links). If some *geometrical* criterion is satisfied, we are sure that the sequent is provable. Girard noticed that in fact, locations in the net, and links between them are sufficient to identify a proof, exactly as if we were getting rid of formulae! (*"everything is at work without logic!"*). This provides a basis for locativity, which opens the way to Ludics. If we ignore formulae, we can only reason on locations (*loci*).

5.3 Designs

By getting rid of formulae, we may formulate the previous rules entirely in terms of addresses (the *loci*) where these formulae were anchored. Loci are sequences of *biases* (or integers). Then emerge two main rules:

- **Positive rule**

$$\frac{\cdots \qquad \xi.i \vdash \Delta_i \qquad \cdots}{\vdash \Delta, \xi}\,(\xi, I)$$

where I may be empty and for every indexes $i, j \in I$ ($i \neq j$), Δ_i and Δ_j are disconnected and every Δ_i is included in Δ.

- **Negative rule**

$$\frac{\cdots \qquad \vdash \xi.I, \Delta_I \qquad \cdots}{\xi \vdash \Delta}\,(\xi, \mathcal{N})$$

where $\mathcal{N}$ is a possibly empty or infinite set of ramifications such that for all $I \in \mathcal{N}$, Δ_I is included in Δ.

Of course, the removal of formulae apparently deprives us of rules which make explicit use of formulae, that is mainly the axiom rule and the cut rule. Actually, the cut rule is *externalized* : the cut is simply a coincidence of loci with opposite polarities and identity will be expressed by the so called $\mathcal{F}ax$ (see 5.5).
In Ludics, we make the assumption that a proof attempt may be stopped at any arbitrary stage. That corresponds to the use of the *daimon* rule:

$$\frac{}{\vdash \Delta}\,\dagger$$

This rule is of course a *paralogism* when compared with axiom rules in Hypersequentialized Sequent Linear Logic since taken litterally it would say that every positive sequent is derivable. But its interpretation is from now on different, it only means that we don't go further in an argumentation. Moreover, the admission of this rule shows the need to embed proofs inside a more general class of objects, some of which are simply

not proofs at all, and hence often called *paraproofs*).
A *design* is a tree of forks built by means of these three rules. Its *basis* is the fork at its bottom. But there is another way to see a design, since a proof process may be also seen as a sequence of negative and positive steps in a game. It is as a set of *possible plays*. These plays are called *chronicles*. A chronicle may be built from a design according to the previous definition. Starting from the bottom, we record all the branches and their sub-branches. In order to correspond to a true design, these chronicles must satisfy some conditions (coherence, propagation, positivity, totality).

5.4 Interaction

Considering two designs of bases of different polarities, interaction consists in a coincidence of two loci in dual position in these bases. This creates a dynamics of rewriting of the cut-net made by the designs, called, as usual, *normalization*. We sum up this process as follows: the cut link is duplicated and propagates over all immediate *subloci* of the initial cut-*locus* as long as the action anchored on the positive fork containing the cut-locus corresponds to one of the actions anchored on the negative one. The process terminates either when the positive action anchored on the positive cut-fork is the *daïmon*, in which case we obtain a design with the same basis as the initial cut-net, or when it happens that in fact, no negative action corresponds to the positive one. In the later case, the process fails (or *diverges*). The process may not terminate since designs are not necessarily finite objects.
When the normalization between two designs $\mathcal{D}$ and $\mathcal{E}$ (respectively based on $\vdash \xi$ and $\xi \vdash$) succeeds, the designs are said to be *orthogonal*, and we note: $\mathcal{D} \perp \mathcal{E}$. In this case, normalization ends up on the particular design :

$$\frac{}{\vdash}\ \dagger$$

Let $\mathcal{D}$ be a design, $\mathcal{D}^{\perp}$ denotes the set of all its orthogonal designs. It is then possible to compare two designs according to their counter-designs. Moreover the separation theorem [Girard 01] ensures that a design is exactly defined by its orthogonal: if $\mathcal{D}^{\perp} = \mathcal{E}^{\perp}$ then $\mathcal{D} = \mathcal{E}$.

5.5 Infinite designs

Infinite designs are useful. Some of them may be recursively defined. It is the case of $\mathcal{F}ax$, which is defined as follows:

$$Fax_{\xi,\xi'} = \cfrac{...\quad \cfrac{...\ \cfrac{Fax_{\xi'_i,\xi_i}}{\xi' \star i \vdash \xi \star i}\ ...}{\vdash \xi \star J, \xi'}\ (+, \xi', J) \quad ...}{\xi \vdash \xi'}\ (-, \xi, \mathcal{P}_f(\mathbb{N}))$$

At the first (negative) step, the negative *locus* is distributed over all the finite subsets of $\mathbb{N}$, then for each set of addresses (relative to some J), the positive locus ξ' is chosen and gives rise to a subaddress $\xi' \star i$ for each $i \in J_k$, and the same machinery is relaunched for the new loci obtained.

5.6 Behaviours

One of the main virtues of this ”deconstruction” is to help us rebuilding Logic.

- Formulae are now some sets of designs. They are exactly those which are closed (or stable) by interaction, that is those which are equal to their *bi-orthogonal*. Technically, they are called *behaviours*.
- The usual connectives of Linear Logic are then recoverable, with the very nice property of *internal completeness*. That is : the bi-closure is useless for all linear connectives. For instance, every design in a behaviour $\mathbf{C} \oplus \mathbf{D}$ may be obtained by taking either a design in $\mathbf{C}$ or a design in $\mathbf{D}$.
- Finally, *proofs* will be now designs satisfiying some properties, in particular that of not using the daïmon rule.

References

[Andréoli 92] J.-M. Andréoli *Logic Programming with Focusing Proofs in Linear Logic*, The Journal of Logic and Computation, 2, 3, pp. 297-347, 1992,

[Asher & Lascarides 98] N. Asher & A. Lascarides *Questions in dialogue*, Linguistics and Philosophy, vol 21, 1998, pp 237–309,

[Beyssade & Marandin 06] C. Beyssade and J-M. Marandin *The speech act assignment problem revisited*, CSSP proceedings, http://www.cssp.cnrs.fr, 2006,

[Brandom 00] R. Brandom *Articulating reasons. An introduction to Inferentialism*, The President and Fellows of Harvard College, 2000,

[Bras et al. 02] M. Bras, P. Denis, P. Muller, L. Prévot, L. Vieu *Une approche sémantique et rhétorique du dialogue*, Traitement Automatique des Langues, vol 43, n^o 2, 2002, pp 43–71,

[Curien 04] Pierre-Louis Curien *Introduction to linear logic and ludics, part I and II*, to appear, downloadable from http://www.pps.jussieu.fr/ curien/LL-ludintroI.pdf,

[Ducrot 1984] O. Ducrot, *Le dire et le dit*, Editions de Minuit, Paris, 1984.

[Girard 99] J.-Y. Girard *On the Meaning of Logical Rules-I* in *Computational Logic*, U. Berger and H. Schwichtenberg eds. Springer-Verlag, 1999,

[Girard 01] J.-Y. Girard *Locus Solum* Mathematical Structures in Computer Science 11, pp. 301-506, 2001,

[Girard 03] J.-Y. Girard *From Foundations to Ludics* Bulletin of Symbolic Logic 09, pp. 131-168, 2003,

[Girard 06] J.-Y. Girard *Le Point Aveugle*, vol. I , II, Hermann, Paris, 2006,

[Lecomte-Quatrini10] A. Lecomte and M. Quatrini *Pour une étude du langage via l'interaction : dialogues et sémantique en Ludique*, Mathématique et Sciences Humaines, n^{o}189, 2010,

[Walton 00] D. Walton *The place of dialogue theory in logic, computer science and communication studies* Synthese 123: pp 327-346, 2000