On a particular class of codes meeting the Griesmer bound

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(joint work with P. Govaerts)

The Griesmer bound states that if there exists an \([n, k, d; q]\) code for given values of \(k, d, q\), then \(n \geq \sum_{i=0}^{d-1} \left\lceil \frac{i}{q-1} \right\rceil = g_i(k, d)\), where \(\lceil x \rceil\) denotes the smallest integer greater than or equal to \(x\).

For given values of \(k, d\) and \(q\), the question arises whether there exists a linear \([n, k, d; q]\) code with length \(n = g_i(k, d)\). This coding-theoretical problem can be translated into a geometrical problem on minihypers in projective spaces.

An \([f, m; k-1, q]\)-minihyper \(F\) is a set of \(f\) points of the projective space \(PG(k-1, q)\) intersecting every hyperplane of \(PG(k-1, q)\) in at least \(m\) points, and there is a hyperplane intersecting \(F\) in exactly \(m\) points [6].

The existence of \([n, k, d; q]\) codes, with \(d = q^{i+1} - \sum_{i=0}^{d-2} \epsilon_i q^i\), \(0 \leq \epsilon_i \leq q-1\), meeting the Griesmer bound, is equivalent to the existence of \(\{\sum_{i=0}^{d-2} \epsilon_i q^i, \sum_{i=0}^{d-2} \epsilon_i (q^i - 1) / (q - 1); k - 1, q\}\)-minihypers.

The classical examples of minihypers of the latter description are pairwise disjoint unions of \(\epsilon_0\) points, \(\epsilon_1\) lines, \ldots, \(\epsilon_{i-2}\) spaces \(PG(k-2, q)\).

For \(\sum_{i=0}^{d-2} \epsilon_i \leq \sqrt{q}\), Hamada, Helleseth and Maekawa [4, 5] proved that all \(\{\sum_{i=0}^{d-2} \epsilon_i(q^{i+1} - 1) / (q - 1), \sum_{i=0}^{d-2} \epsilon_i(q^i - 1) / (q - 1); k - 1, q\}\)-minihypers are of this type.

If \(\sum_{i=0}^{d-2} \epsilon_i \geq \sqrt{q} + 1\), then other examples of minihypers of the considered type occur; minihypers containing subgeometries \(PG(\ell, \sqrt{q})\).

For \(\sum_{i=0}^{d-2} \epsilon_i \leq \frac{2}{\sqrt{q}} - 1\) and \(q\) large, Ferret and Storme [1] proved that a \(\{\sum_{i=0}^{d-2} \epsilon_i(q^{i+1} - 1) / (q - 1), \sum_{i=0}^{d-2} \epsilon_i(q^i - 1) / (q - 1); k - 1, q\}\)-minihyper consists of the pairwise disjoint union of subspaces over \(GF(q)\) and at most one subgeometry over \(GF(\sqrt{q})\).

In the particular case of a \(\{\epsilon(q^{t+1} - 1) / (q - 1), \epsilon(q^t - 1) / (q - 1); k - 1, q\}\)-minihyper, further improvements were obtained by Govaerts and Storme [2, 3].

They proved that such a minihyper is the union of: (1) \(\epsilon\) pairwise disjoint spaces \(PG(t, q)\) when \(\epsilon < (q + 3) / 2\) for \(q\) an odd prime, or \(\epsilon \leq q^{2/3}\) when \(q\) is a non-square, and (2) a pairwise disjoint union of subspaces \(PG(t, q)\) and subgeometries \(PG(2t + 1, \sqrt{q})\) when \(\epsilon < q^{5/8} / \sqrt{2} + 1\) when \(q\) is a square.
References


