On a particular class of codes meeting the Griesmer bound

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(joint work with P. Govaerts)

The *Griesmer bound* states that if there exists an $[n,k,d;q]$ code for given values of k,d,q, then $n \geq \sum_{i=0}^{k-1} \lceil \frac{d}{q^i} \rceil = g_q(k,d)$, where $\lceil x \rceil$ denotes the smallest integer greater than or equal to x.

For given values of k,d and q, the question arises whether there exists a linear $[n,k,d;q]$ code with length $n = g_q(k,d)$. This coding-theoretical problem can be translated into a geometrical problem on *minihypers in projective spaces*.

An $\{f,m;k-1,q\}$-*minihyper* F is a set of f points of the projective space $PG(k-1,q)$ intersecting every hyperplane of $PG(k-1,q)$ in at least m points, and there is a hyperplane intersecting F in exactly m points [6].

The existence of $[n,k,d;q]$ codes, with $d = q^{k-1} - \sum_{i=0}^{k-2} \epsilon_i q^i$, $0 \leq \epsilon_i \leq q-1$, meeting the Griesmer bound, is equivalent to the existence of $\{\sum_{i=0}^{k-2} \epsilon_i (q^{i+1}-1)/(q-1), \sum_{i=0}^{k-2} \epsilon_i (q^i - 1)/(q-1); k-1, q\}$-minihypers.

The classical examples of minihypers of the latter description are pairwise disjoint unions of ϵ_0 points, ϵ_1 lines, ..., ϵ_{k-2} spaces $PG(k-2,q)$.

For $\sum_{i=0}^{k-2} \epsilon_i \leq \sqrt{q}$, Hamada, Helleseth and Maekawa [4, 5] proved that all $\{\sum_{i=0}^{k-2} \epsilon_i (q^{i+1}-1)/(q-1), \sum_{i=0}^{k-2} (q^i-1)/(q-1); k-1, q\}$-minihypers are of this type.

If $\sum_{i=0}^{k-2} \epsilon_i \geq \sqrt{q}+1$, then other examples of minihypers of the considered type occur; minihypers containing subgeometries $PG(i,\sqrt{q})$.

For $\sum_{i=0}^{k-2} \epsilon_i \leq 2\sqrt{q}-1$ and q large, Ferret and Storme [1] proved that a $\{\sum_{i=0}^{k-2} \epsilon_i (q^{i+1}-1)/(q-1), \sum_{i=0}^{k-2} \epsilon_i (q^i-1)/(q-1); k-1, q\}$-minihyper consists of the pairwise disjoint union of subspaces over $GF(q)$ and at most one subgeometry over $GF(\sqrt{q})$.

In the particular case of a $\{\epsilon(q^{t+1}-1)/(q-1), \epsilon(q^t-1)/(q-1); k-1, q\}$-minihyper, further improvements were obtained by Govaerts and Storme [2, 3].

They proved that such a minihyper is the union of: (1) ϵ pairwise disjoint spaces $PG(t,q)$ when $\epsilon < (q+3)/2$ for q an odd prime, or $\epsilon \leq q^{2/3}$ when q is a non-square, and (2) a pairwise disjoint union of subspaces $PG(t,q)$ and subgeometries $PG(2t+1,\sqrt{q})$ when $\epsilon < q^{5/8}/\sqrt{2}+1$ when q is a square.

References

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