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Abstract: A collection of n not necessarily distinct points in the projective space $^{k-1}$ over $_q$, which are not contained in a hyperplane, gives rise to a nondegenerate projective system in the sense of Tsfasman and Vlăduţ. Such a projective system corresponds naturally to a linear $[n, k]_q$-code (cf. [10]). Some natural sources for projective systems are the sets of $_q$-rational points of projective algebraic varieties defined over $_q$, and in this way a useful link between higher dimensional varieties and linear codes is established.

We shall consider, in particular, the codes obtained from the classical Grassmann varieties with their Plücker embedding and more generally, the Schubert varieties in Grassmannians. The resulting Grassmann code has been studied by Ryan [6, 7], Nogin [4] and by Ghorpade and Lachaud [3]. A generalization to Schubert codes was proposed in [3] and a conjecture due to the first author concerning the minimum distance of Schubert codes $C_\alpha(\ell, m)$ was also made there. Recently, this conjecture has been proved in the case of $\ell = 2$ by Hao Chen [2]. In a related development, Rodier [5] has considered codes associated to flag varieties. However, the general case of the abovementioned conjecture as well as some basic questions concerning the length and the dimension of the Schubert codes remain unanswered. We shall briefly review these developments and present some results of a recent joint work with M. Tsfasman, concerning the basic parameters of Schubert codes. Related open problems and connections with certain general conjectures of Tsfasman and the results of Serre [8], Sørrensen [9] and Boguslavsky [1] will also be discussed.

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